Math 213 Quiz # 3 Solutions

That guy

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Hi. I'm that guy you all love to hate. Here are the solutions.

1. If $f(x) = \frac{1}{1+x}$, then $f(0) = 1$, $f'(x) = \frac{-1}{(x+1)^2}$, and $f''(x) = \frac{2}{(x+1)^3}$, so $f'(0) = -1$ and $f''(0) = 2$. So the Maclaurin expansion is $1 - x + x^2$ (recall that there are factors of $1/0! = 1, 1/1! = 1$ and $1/2! = 1/2$ on each term, doing some cancelling on the degree 2 term).

2. Using the ratio test with $a_n = \frac{nx^n}{3^n}$, we have (all limits are as $n \to \infty$):

$$\lim \left| \frac{\frac{(n+1)x^{n+1}}{3^{n+1}}}{\frac{nx^n}{3^n}} \right| = \lim \left| \frac{x(n+1)}{3n} \right| = \frac{|x|}{3}.$$

   So the radius of convergence is 3, and plugging in $x = \pm 3$ yields a divergent series by the test for divergence. Thus, the interval of convergence is $(-3, 3)$.

3. Using the ratio test with $a_n = \frac{nx^n}{3^n}$, we have (all limits are as $n \to \infty$):

$$\lim \left| \frac{\frac{(-1)^{n+1}x^{2n+2}}{(n+1)^2 2^{n+1}}}{\frac{(-1)^n x^{2n}}{n^2 2^n}} \right| = \lim |x^2| \frac{1}{2} \frac{n^2}{(n+1)^2} = \frac{|x|^2}{2}.$$

   So if $\frac{|x|^2}{2} < 1$, then $|x|^2 < 2$, and so $|x| < \sqrt{2}$, and the radius of convergence is $\sqrt{2}$. One plugs in $x = \pm\sqrt{2}$ and gets the series $\sum \frac{(-1)^n}{n^2}$, which absolutely converges. So the interval of convergence is $[-\sqrt{2}, \sqrt{2}]$.