# Math 213 Quiz # 2 Solutions

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February 24, 2008

*HI EVERYBODY! It's Chris Griffin here to tell you the solutions to the recent quiz # 2. HOPE YOU LIKE IT! Please see the end of the document for other general remarks about this quiz.*

1. (a) This sequence converges to 2. This can be seen by factoring out an $n^2$ on the numerator or denominator, or by L'Hospital's Rule. Below, we use L'Hospital's Rule:

$$\lim_{n\to\infty} \frac{2n^3+1}{n^3-6n+1} = \lim_{x\to\infty} \frac{2x^3+1}{x^3-6x+1} = \lim_{x\to\infty} \frac{6x^2}{3x^2-6} = \lim_{x\to\infty} \frac{12x}{6x} = \lim_{x\to\infty} 2 = 2.$$

(b) We use the squeeze theorem, and note that $0 \leq \sin^2(n) \leq 1$. So

$$0 \leq \frac{\sin^2(n)}{n} \leq \frac{1}{n}.$$

Now since both $0 \to 0$ and $\frac{1}{n} \to 0$, we conclude that $\frac{\sin^2(n)}{n} \to 0$.

(c) We note that $\frac{n!}{(n+1)!} = \frac{1}{n+1} \to 0$.

2. (a) This series diverges by the test for divergence:

$$\lim_{n\to\infty} \frac{n+2}{n} = 1 \neq 0.$$

(b) We use the limit comparison test with the series $b_n = \frac{1}{n}$, and $\sum \frac{1}{n}$ is known to diverge since it is a $p$-series, with $p = 1$. Then

$$\lim_{n\to\infty} \frac{1/(n+1)}{1/n} = \lim_{n\to\infty} \frac{n}{n+1} = 1, 0 < 1 < \infty.$$

So the limit comparison test applies, and since $\sum \frac{1}{n}$ diverges, we have that $\sum \frac{1}{n+1}$ diverges.

*Remark. A surprising number of you made a direct comparison of the form* $\frac{1}{n+1} \leq \frac{1}{n}$*, which is true. But since* $\sum \frac{1}{n}$ *diverges, this comparison is of no use. In a certain sense, all it would say is that* $\sum \frac{1}{n+1} \leq \infty$*, which does not force it to either converge or diverge.*

(c) This is a geometric series, as $e^{-n} = \left(\frac{1}{e}\right)^n$. So since $-1 < \frac{1}{e} < 1$, we conclude that the given series converges.

(d) We use the limit comparison test with $b_n = \frac{1}{n}$, which as mentioned before, $\sum \frac{1}{n}$ is a divergent $p$-series. We check

$$\lim_{n\to\infty} \frac{n/(n^2 - 4n + 18)}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2 - 4n + 18} = 1.$$

So the limit comparison test applies, and the given series diverges.

(e) We use the limit comparison test with $b_n = \left(\frac{3}{4}\right)^n = \frac{3^n}{4^n}$. The series $\sum \left(\frac{3}{4}\right)^n$ is a convergent geometric series. Then we check that the limit comparison test applies, first by studying the quotient $\frac{a_n}{b_n}$:

$$\frac{\frac{3^n}{4^n-1}}{\frac{3^n}{4^n}} = \frac{4^n}{4^n - 1}.$$

Now we apply L'Hospital's rule to see that

$$\lim_{n\to\infty} \frac{4^n}{4^n - 1} = \lim_{x\to\infty} \frac{4^x}{4^x - 1} = \lim_{x\to\infty} \frac{(\ln 4)4^x}{(\ln 4)4^x} = \lim_{x\to\infty} 1 = 1.$$

*Other general remarks.* Many of you seemed to confuse what question was being asked. Question number 1 asked you if a *sequence* converged, while question number 2 asked you to determine if a *series* converged. So Corey saw a lot of curious answers, mostly on question 1. So he suggests that if you made these errors, to be sure you know what the question is asking, and thus, that you know what information is needed. In addition, several of you made remarks about what the series converges to, even though that information wasn't asked. Several of you claimed that the geometric series in part (c) of question 2 converged to 0. Yes, it converges, but how could it possibly converge to 0 if all of the terms are greater than 0? (the confusion, Corey thinks, is confusing the value that the sequence $e^{-n}$ converges to with the value that the series $\sum e^{-n}$ converges to). Of course, regardless of this, it's never a good idea to volunteer more information on a quiz or test than exactly what is asked for. Of course, your justification must be there (after all, the directions asked for your justification), but rarely would Corey ask what a series converges to on a quiz covering this material. This will change, of course, as we develop more tools for how to evaluate series, but Corey urges you to read the directions.