Math 213 Exam # 1 solutions!

Scooby Doo!!!!

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Hi everyone! Here are the solutions to the first exam! Rock on!

1. We use the disk (washer) method, with $R = x^2$ and $r = x^3$. Then the volume is

$$\begin{aligned} volume &= \pi \int_0^1 ((x^2)^2 - (x^3)^2)dx \\ &= \pi \int_0^1 x^4 - x^6 dx \\ &= \pi (x^5/5 - x^7/7)|_0^1 \\ &= \pi (1/5 - 1/7) \\ &= \frac{2\pi}{35}. \end{aligned}$$

2. This region I can't draw, since I'm just a dog. But I can describe it. It's the region above the parabola that is capped off at $y = 6$. As we try to set up the integral to go around the x-axis, I think the shell method would be easier, since each of the heights are straightforward to find. The disk method is possible, but it requires knowing x as a function of y.

3. We use the shell method, with the height $h(x) = \frac{1}{x}$, between $x = 1$ and $x = 4$. Then

$$\begin{aligned} volume &= 2\pi \int_1^4 x \frac{1}{x} dx \\ &= 2\pi \int_1^4 dx \\ &= 2\pi \cdot 3 \\ &= 6\pi. \end{aligned}$$

4. We compute

$$y' = \frac{3}{2}(4 - x^{2/3})^{1/2} \cdot (-\frac{2}{3}x^{-1/3}) = -x^{-1/3}(4 - x^{2/3})^{1/2}$$

So

$$\sqrt{1 + (y')^2} = \sqrt{1 + \frac{4 - x^{2/3}}{x^{2/3}}} = \sqrt{\frac{4}{x^{2/3}}} = 2x^{-1/3}.$$

So the arc length is given by the integral:

$$\int_0^8 2x^{-1/3} dx = 3x^{2/3}|_0^8 = 3 * (8^{2/3}) - 3 * (0) = 12.$$

5. With $y = \sqrt{x+1}$, we have

$$\sqrt{1 + (y')^2} = \sqrt{1 + \frac{1}{4(x+1)}} = \sqrt{\frac{4(x+1)+1}{4(x+1)}} = \frac{1}{2\sqrt{x+1}}\sqrt{4x+5}$$

So the surface area is given by

$$2\pi \int_0^1 \sqrt{x+1} \cdot \frac{1}{2\sqrt{x+1}}\sqrt{4x+5} = \pi \int_0^1 \sqrt{4x+5}dx = \frac{\pi}{6}(4x+5)^{3/2}|_0^1 = \frac{\pi}{6}(9^{3/2} - 5^{3/2}).$$

6. Using $y = \sqrt{r^2 - x^2}$, remember that r is a constant, and that using the chain rule, the derivative $y' = \frac{-x}{\sqrt{r^2-x^2}}$. Then $1 + (y')^2 = 1 + \frac{x^2}{r^2-x^2} = \frac{r^2}{r^2-x^2}$. Then according to our formula, the surface area is:

$$2\pi \int_{-r}^r y\sqrt{1 + (y')^2}dx = 2\pi \int_{-r}^r \sqrt{r^2 - x^2} \cdot \frac{r}{\sqrt{r^2 - x^2}}dx = 2\pi \int_{-r}^r r dx = 4\pi r^2.$$