Quiz # 2 Solutions

March Madness!!!!

March 16, 2010

Here are the solutions to the second quiz. Enjoy!

1. Notice that $\ln\left(\frac{2x+1}{3x+4}\right) = \ln(2x+1) - \ln(3x+4)$. So

$$\frac{d}{dx}\left[\ln\left(\frac{2x+1}{3x+4}\right)\right] = \frac{d}{dx}\left[\ln(2x+1) - \ln(3x+4)\right] = \frac{2}{2x+1} - \frac{3}{3x+4}.$$

2. Let $u = e^x$. Then $du = e^x dx$, and $\int e^x \sec^2 e^x dx = \int \sec^2 u du = \tan u + C = \tan e^x + C$.

3. We consider the option that this is an arcsin type integral, and factor out the 4 from the denominator:

$$\int \frac{1}{\sqrt{4-x^2}}dx = \frac{1}{2}\int \frac{1}{\sqrt{1-(x/2)^2}}dx.$$

We let $u = x/2$, so that $du = \frac{1}{2}dx$, $2du = dx$, and

$$\frac{1}{2}\int \frac{1}{\sqrt{1-(x/2)^2}}dx = \frac{1}{2}\int \frac{2du}{\sqrt{1-u^2}} = \int \frac{1}{\sqrt{1-u^2}}du = \arcsin(u)+C = \arcsin(x/2)+C.$$

4. We use integration by parts. Let $u = e^{-x}, dv = \sin x dx$. Then $du = -e^{-x}dx, v = -\cos x$. Then

$$\int e^{-x}\sin x dx = -e^{-x}\cos x - \int e^{-x}\cos x dx.$$

We use integration by parts again. Let $u = e^{-x}, dv = \cos x dx$, so that $du = -e^{-x}dx, v = \sin x$. Then

$$\begin{aligned} \int e^{-x}\sin x dx &= -e^{-x}\cos x - \int e^{-x}\cos x dx \\ &= -e^{-x}\cos x - \left[e^{-x}\sin x - \int -e^{-x}\sin x dx\right] \\ &= -e^{-x}\cos x - e^{-x}\sin x - \int e^{-x}\sin x dx \end{aligned}$$

Adding $\int e^{-x}\sin x dx$ to both sides and dividing by 2 gives us the result of

$$\int e^{-x}\sin x dx = \frac{1}{2}\left(-e^{-x}\cos x - e^{-x}\sin x\right) + C.$$

5. We notice that we can add and subtract 2 from the numerator, and complete the square to see that:

$$\frac{2x}{x^2+2x+5} = \frac{2x+2}{x^2+2x+5} - \frac{2}{x^2+2x+5} = \frac{2x+2}{x^2+2x+5} - \frac{2}{(x+1)^2+4}.$$

Using $u = x^2 + 2x + 5$, and $du = (2x+2)dx$, the first integral is

$$\int \frac{2x+2}{x^2+2x+5}dx = \int \frac{du}{u} = \ln|x^2+2x+5| + C.$$

We treat the second integral as an arctangent integral, and factor a 4 out of the denominator:

$$\frac{2}{x^2+2x+5} = \frac{1}{2}\left(\frac{1}{1+\left(\frac{x+1}{2}\right)^2}\right)$$

So if $u = \frac{x+1}{2}$, then $du = \frac{1}{2}dx$, and so

$$\int \frac{2}{x^2+2x+5}dx = \int \frac{1}{2}\left(\frac{1}{1+\left(\frac{x+1}{2}\right)^2}\right)dx = \int \frac{1}{1+u^2}du = \arctan(u)+C = \arctan\left(\frac{x+1}{2}\right)+C.$$

Putting these together gives us

$$\int \frac{2x}{x^2+2x+5}dx = \ln|x^2+2x+5| - \arctan\left(\frac{x+1}{2}\right) + C.$$