Quiz # 1 Solutions

The Invisible Man

October 16, 2007

Hi kids, this is the invisible man. I don't have a picture because, well, I'm invisible! Here are the solutions to the quiz # 1. ROCK ON!

1. The function y that we are looking for must be an antiderivative of $\cos x + 2$. So $y = \sin x + 2x + C$ for some C. Since it is given that $y(0) = 1$, we have

$$1 = y(0) = \sin 0 + 2 \cdot 0 + C, \text{ so } C = 1, \text{ and } y = \sin x + 2x + 1\,.$$

2. (a) $\int x^4 + 2x + 1dx = \frac{1}{5}x^5 + x^2 + x + C$.
   (b) $\int \frac{x^6-3}{x^4}dx = \int x^2 - \frac{3}{x^4}dx = \frac{1}{3}x^3 - \frac{3}{-3}x^{-3} + C = \frac{1}{3}x^3 + x^{-3} + C$.

3. (a) Each of the widths are $\Delta x = \frac{2}{4} = \frac{1}{2}$. Using right-hand sums, we have the inputs as $x_1 = -1/2, x_2 = 0, x_3 = 1/2, x_4 = 1$. The outputs are $f(x_1) = \sqrt{3/4}, f(x_2) = 1, f(x_3) = \sqrt{3/4}, f(x_4) = 0$. So the approximation is

$$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x = \sqrt{3/4}\cdot\frac{1}{2} + 1\cdot\frac{1}{2} + \sqrt{3/4}\cdot\frac{1}{2} + 0\cdot\frac{1}{2} = \frac{1}{2}(1+\sqrt{3}).$$

   (b) The actual area would be one-half of $\pi(1)^2$, or $\frac{\pi}{2}$.

4. Our widths are $\Delta x = \frac{3}{n}$. The inputs $x_i = 0 + i\frac{3}{n} = \frac{3i}{n}$. Our outputs are $f(x_i) = \frac{6i}{n} + 1$, and so the Riemann sum is:

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}(\frac{6i}{n} + 1)\frac{3}{n} = \sum_{i=1}^{n}(\frac{18i}{n^2} + \frac{3}{n}) = \frac{18}{n^2}\frac{n(n+1)}{2} + 3\,.$$

   So the actual area $\int_0^3 2x + 1dx = \lim_{n\to\infty}\left[\frac{18}{n^2}\frac{n(n+1)}{2} + 3\right] = 12.$