Midterm #2 Solutions

The Sobe Life Water Lizards

November 25, 2007

Hi everyone! We're the Sobe Lizards, and we're here to give you the solutions to the recent midterm. We've also put in some commentary for each problem to help those people who made some common errors. We strongly suggest that you work hard to understand where you made your mistakes... most mistakes were a combination of algebraic errors and basic calculus errors. These errors are really bad news, since they effect so much of what the exam is about, and in addition what is to come. Unfortunately, this means that partial credit was impossible to award in many situations since many of these errors made the problem much easier, avoiding what Corey was trying to ask altogether. So be sure to determine why it is you missed the points, and Corey is happy to help, of course, whenever he can. A lot of the class did quite well, and there were several perfect scores! But Corey wants everyone to do well, and wants to make sure everyone knows how they can correct these mistakes before the final exam. So we hope you enjoy these solutions! Rock on!

1. Using the first fundamental theorem of Calculus, we have:

$$\frac{d}{dx}\int_1^{e^{3x}} \sqrt{t^6+1}\,dt = \sqrt{(e^{3x})^6+1}\cdot\frac{d}{dx}e^{3x} = \sqrt{(e^{3x})^6+1}\cdot 3e^{3x}.$$

Lots of people missed points on this one, for what are a variety of reasons. There were many people who tried to antidifferentiate this function. . . I have no idea how to actually do this, which is okay, since Corey didn't ask for that, and has never asked you to actually preform an antidifferentiation for problems of this sort. The key is to use the fundamental theorem of calculus to avoid these problems of antidifferentiation. For those that are still unsure, review the Fundamental Theorem of Calculus section, and the class notes.

2. For this problem, Corey wanted to know if you were able to manipulate natural logarithm expressions, and display knowledge of how to differentiate natural logarithm expressions. Most people were okay at this, but some others made the canonical mistake of thinking $\ln(a+b) = \ln(a) + \ln(b)$. Some others forgot to differentiate their expression altogether, which is another way people lost points. In addition, I want everyone to know that

$$\frac{x^2}{1+x^2} \neq \frac{x^2}{1} + \frac{x^2}{x^2}.$$

For some reason, this is a mistake that happens more than any other mistake I think I've ever seen, and as we mentioned at the beginning of this document, mistakes such as these and the others already mentioned really get quite far from what Corey was trying to ask. There was also some confusion as to what logarithm rules are valid, and what aren't. Rather than listing what isn't true, we'll just write out the solution. We have that $\ln(\frac{x^2}{1+x^2}) = \ln x^2 - \ln(1+x^2) = 2\ln x - \ln(1+x^2)$. So

$$\begin{aligned} \tfrac{d}{dx}\ln\left(\tfrac{x^2}{1+x^2}\right) &= \tfrac{d}{dx} 2\ln x - \ln(1+x^2) \\ &= \tfrac{2}{x} - \tfrac{d}{dx}\ln(1+x^2) \\ &= \tfrac{2}{x} - \tfrac{1}{1+x^2}\cdot 2x\,. \end{aligned}$$

3. Using u-substitution, we set $u = 1+x^3$, and so $du = 3x^2dx$. Then

$$\int \frac{3x^2}{1+x^3}dx = \int \frac{du}{u} = \ln|u| + C = \ln|1+x^3| + C\,.$$

4. We let $u = x-1$. Then we have $du = dx$ and $u+1 = x$. So we substitute and get

$$\int x\sqrt{x-1}dx = \int (u+1)\sqrt{u}\;du = \int u^{3/2} + u^{1/2}du$$

$$= \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C.$$

5. The average value of this function on this interval is given as $\frac{1}{3-0}\int_0^3 \frac{x}{x+1}dx$. So we compute the integral, and use u-substitution, with $u = x+1$, so that $du = dx$. In addition, we have $u-1 = x$, so

$$\int \frac{x}{x+1}dx = \int \frac{u-1}{u}du = \int 1 - \frac{1}{u}du = u - \ln|u| + C = x + 1 - \ln|x+1|.$$

When we evaluate between 0 and 3, and multiply by $\frac{1}{3}$, we get the average value as

$$\frac{1}{3}\int_0^3 \frac{x}{x+1}dx = \frac{1}{3}[x + 1 - \ln|x+1||_0^3] = \frac{1}{3}[(4 - \ln 4) - (1 - \ln 1)] = \frac{1}{3}(3 - \ln 4).$$

6. We use $u = \sin x$, then $du = \cos x dx$, and

$$\int_0^\pi \cos x e^{\sin x} dx = \int_{x=0}^\pi e^u du = e^u|_{x=0}^\pi = e^{\sin x}|_0^\pi = e^{\sin\pi} - e^{\sin 0} = 1 - 1 = 0.$$

7. We use u-substitution, with $u = 1 + e^x$. Then $du = e^x dx$, and so $e^{2x}dx = e^x e^x dx = e^x du$. We notice that $u - 1 = e^x$, so the numerator $e^{2x}dx = (u-1)du$. Now we have

$$\int \frac{e^{2x}}{1+e^x}dx = \int \frac{u-1}{u}du = \int 1 - \frac{1}{u}du = u - \ln|u| + C = (1 + e^x) - \ln|1 + e^x| + C.$$