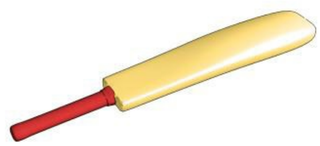
Midterm # 1 solutions

A cartoon cricket bat

November 2, 2007



Hi kids! Here are the solutions to the first midterm. Enjoy!!!

1. (a) We break the interval up into 4 pieces, each with width $\Delta x = \frac{1}{4} = .25$. So, for a right approximation, we would have inputs $x_1 = 1.25, x_2 = 1.5, x_3 = 1.75, x_4 = 2$. So $f(x_1) = 3.5, f(x_2) = 4, f(x_3) = 4.5, f(x_4) = 5$. So the approximate area with a right hand estimate is

$$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x = 3.5 \cdot .25 + 4 \cdot .25 + 4.5 \cdot .25 + 5 \cdot .25$$
$$= .25(3.5 + 4 + 4.5 + 5) = .25(17) = \frac{17}{4}.$$

The right hand approximation is the same except $x_0 = 1$, $f(x_0) = 3$, and the approximation is

$$f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x = 3 \cdot .25 + 3.5 \cdot .25 + 4 \cdot .25 + 4.5 \cdot .25$$
$$= \frac{15}{4}.$$

(b) Using the limit process, we have $\Delta x = \frac{2-1}{n} = \frac{1}{n}$. We also have $x_k = 1 + \frac{k}{n}$, and $f(x_k) = 2x_k + 1 = \frac{2k}{n} + 3$. So the Riemann sum is $\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}(\frac{2k}{n} + 3)\frac{1}{n} = \sum_{k=1}^{n} \frac{2k}{n^2} + \frac{3}{n}$. Now $\sum_{k=1}^{n} \frac{3}{n} = 3$, and $\sum_{k=1}^{n} \frac{2k}{n^2} = \frac{2}{n^2} \sum_{k=1}^{n} k = \frac{n+1}{n}$. So

$$\int_1^2 (2x+1)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \lim_{n\to\infty} \frac{n+1}{n} + 3 = 4.$$

(c) We have $\int_1^2 (2x+1)dx = x^2 + x|_1^2 = 6 - 2 = 4$.

2. See the quiz solutions, problem #1.

3. (a) $\sum_{i=1}^{3}(2i+1) = 3+5+7 = 15$.

   (b) $\sum_{i=1}^{4}(i-1) = 0+1+2+3 = 6$.

   (c) According to our formula for $n = 100$, we have $\sum_{k=1}^{100} i = 5050$.

4. (a) We have $\int x^2 - 6x + 1dx = \frac{1}{3}x^3 - 3x^2 + x + C$.

   (b) $\int \cos x - \sin x dx = \sin x + \cos x + C$.

   (c) $\int \frac{\sqrt{x}-1}{\sqrt{x}} dx = \int 1 - x^{-1/2} dx = x - 2x^{1/2} + C$.

   (d) $\int \frac{x^2}{1+x^2} dx = \int \frac{1+x^2-1}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$.

5. (a) $\int_0^4 4x^3 - 2x dx = x^4 - x^2|_0^4 = 256 - 16 = 240$.

   (b) $\int_0^\pi \sin x dx = -\cos x|_0^\pi = (-\cos\pi) - (-\cos 0) = 1 + 1 = 2$.

   Rock on!!!!