

Quiz # 3 Solutions

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Hi everyone! I hope you enjoy these solutions! ROCK ON!

1. Let $u = x$, $dv = e^x dx$. Then $du = dx$, and $v = e^x$. And so we have

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C.$$

2. We will use integration by parts twice. First, we let $u = x^2$, and $dv = \cos x dx$. Then $du = 2x dx$, and $v = \sin x$. So

$$\int x^2 \cos x dx = x^2 \sin x - \int 2x \sin x dx.$$

To integrate $\int 2x \sin x dx$, we let $u = 2x$, and $dv = \sin x dx$. Then $du = 2$, and $v = -\cos x$. So

$$\int 2x \sin x dx = -2x \cos x - \int -2 \cos x dx = -2x \cos x + \int 2 \cos x dx.$$

Plugging that into the above, we get

$$\begin{aligned} \int x^2 \cos x dx &= x^2 \sin x - \int 2x \sin x dx \\ &= x^2 \sin x - (-2x \cos x + \int 2 \cos x dx) \\ &= x^2 \sin x + 2x \cos x - 2 \sin x + C. \end{aligned}$$

3. We choose $u = e^x$, and $dv = \sin x dx$. Then $du = e^x dx$, and $v = -\cos x$. So

$$\int e^x \sin x dx = -e^x \cos x - \int -e^x \cos x dx = -e^x \cos x + \int e^x \cos x dx.$$

To integrate $\int e^x \cos x dx$, we choose $u = e^x$ and $dv = \cos x dx$. Then $du = e^x dx$, and $v = \sin x$. Then

$$\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx.$$

Plugging this into the top, we get

$$\begin{aligned} \int e^x \sin x dx &= -e^x \cos x + (e^x \sin x - \int e^x \sin x dx) \\ &\quad \text{now add } \int e^x \sin x dx \text{ to both sides:} \\ 2 \int e^x \sin x dx &= -e^x \cos x + e^x \sin x, \text{ so} \\ \int e^x \sin x dx &= \frac{1}{2} [-e^x \cos x + e^x \sin x]. \end{aligned}$$

ROCK ON.