

# Quiz # 1 Solutions

The Invisible Man

April 20, 2007

*Hi kids, this is the invisible man. I don't have a picture because, well, I'm invisible! Here are the solutions to the quiz # 1. ROCK ON!*

1. The function  $y$  that we are looking for must be an antiderivative of  $\cos x + 2$ . So  $y = \sin x + 2x + C$  for some  $C$ . Since it is given that  $y(0) = 1$ , we have

$$1 = y(0) = \sin 0 + 2 \cdot 0 + C, \text{ so } C = 1, \text{ and } y = \sin x + 2x + 1.$$

2. (a)  $\int x^4 + 2x + 1 dx = \frac{1}{5}x^5 + x^2 + x + C.$

(b)  $\int \frac{x^6-3}{x^4} dx = \int x^2 - \frac{3}{x^4} dx = \frac{1}{3}x^3 - \frac{3}{-3}x^{-3} + C = \frac{1}{3}x^3 + x^{-3} + C.$

3. (a) Each of the widths are  $\Delta x = \frac{2}{4} = \frac{1}{2}$ . Using right-hand sums, we have the inputs as  $x_1 = -1/2, x_2 = 0, x_3 = 1/2, x_4 = 1$ . The outputs are  $f(x_1) = \sqrt{3/4}, f(x_2) = 1, f(x_3) = \sqrt{3/4}, f(x_4) = 0$ . So the approximation is

$$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x = \sqrt{3/4} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + \sqrt{3/4} \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} = \frac{1}{2}(1 + \sqrt{3}).$$

(b) The actual area would be one-half of  $\pi(1)^2$ , or  $\frac{\pi}{2}$ .

4. Our widths are  $\Delta x = \frac{3}{n}$ . The inputs  $x_i = 0 + i\frac{3}{n} = \frac{3i}{n}$ . Our outputs are  $f(x_i) = \frac{6i}{n} + 1$ , and so the Riemann sum is:

$$\sum_{i=1}^n f(x_i)\Delta x = \sum_{i=1}^n \left(\frac{6i}{n} + 1\right) \frac{3}{n} = \sum_{i=1}^n \left(\frac{18i}{n^2} + \frac{3}{n}\right) = \frac{18}{n^2} \frac{n(n+1)}{2} + 3.$$

So the actual area  $\int_0^3 2x + 1 dx = \lim_{n \rightarrow \infty} \left[ \frac{18}{n^2} \frac{n(n+1)}{2} + 3 \right] = 12.$