

Math 212 Quiz #1 Solutions and Comments

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Hello everyone, Vitamin C here to tell you all about Quiz #1! Overall, the quizzes looked good, I've included a grade breakdown below. As a sensible vitamin, though, I wanted to share with you some of my comments regarding some of the problems so hopefully fewer people make the same mistakes on the midterm next week. So, enjoy! And remember that you can find Vitamin C in many other things besides oranges!

Score Range	# of people within that range
45- ∞	9
40-44	2
35-39	3
30-34	2
$-\infty$ - 29	3

1. It seems as though some people forgot exactly what question number 1 asked for. This dates back to the first few days of class, and is a very important concept. The question asks you to play a game and win. The game goes like this. I'm thinking of a function whose name is y , and I'm giving you two hints as to what it is. The first is that its derivative is $\cos x + 2$. The second hint is that when you plug in 0, you get an output of 1. What's the function? Well, the first hint tells you that it's an antiderivative of $\cos x + 2$, so y must be of the form $y = \sin x + 2x + C$ for some C . Most of you who missed this problem seemed rather to differentiate the equation and get something like $-\sin x$. We're going the other way. The second clue tells you that

$$1 = y(0) = \sin 0 + 2 \cdot 0 + C, \text{ so } C = 1.$$

So, the answer must be $y = \sin x + 2x + 1$.

2. (a) Most people got this one, although without the $+C$ you missed one point:

$$\int x^4 + 2x + 1 dx = \frac{1}{5}x^5 + x^2 + x + C.$$

3. (b) Several people made the mistake of either simplifying this incorrectly, or integrating it incorrectly. Most who integrated incorrectly made the mistake of thinking something was true when it actually isn't. Let me remind all of you that $\int f(x) \cdot g(x) dx \neq (\int f(x) dx) (\int g(x) dx)$. The way I would have simplified it, and integrated it is as follows:

$$\int \frac{x^6 - 3}{x^4} dx = \int \frac{x^6}{x^4} dx - \frac{3}{x^4} dx = \int x^2 dx - 3 \int x^{-4} dx.$$

From there, since $\int x^2 dx = \frac{1}{3}x^3 + C$ and $3 \int x^{-4} dx = \frac{3}{-3}x^{-3} + C = -x^{-3} + C$, the answer is

$$\int \frac{x^6 - 3}{x^4} dx = \frac{1}{3}x^3 - (-x^{-3}) + C = \frac{1}{3}x^3 + x^{-3} + C.$$

4. (a) This problem asked only for an approximation using 4 boxes from $x = 0$ to $x = 1$. Some people who missed points did so because they made it a little more complicated than it needed to be, thus the benefit of the Fundamental Theorem of Calculus. In either approximation, left or right, we'd have

$$\Delta x = \frac{1-0}{4} = \frac{1}{4}, x_i = 0 + i\frac{1}{4} = \frac{i}{4} \text{ for } i = 0, 1, \dots, 4.$$

So, $x_0 = 0, x_1 = \frac{1}{4}, x_2 = \frac{2}{4}, x_3 = \frac{3}{4}, x_4 = 1$. So the left approximation is

$$f(0)\frac{1}{4} + f\left(\frac{1}{4}\right)\frac{1}{4} + f\left(\frac{2}{4}\right)\frac{1}{4} + f\left(\frac{3}{4}\right)\frac{1}{4} = \frac{1}{4} + \sqrt{\frac{15}{16}}\frac{1}{4} + \sqrt{\frac{3}{4}}\frac{1}{4} + \sqrt{\frac{7}{16}}\frac{1}{4} \approx .8739 \dots$$

Similarly, the right approximation is

$$f\left(\frac{1}{4}\right)\frac{1}{4} + f\left(\frac{2}{4}\right)\frac{1}{4} + f\left(\frac{3}{4}\right)\frac{1}{4} + f(1)\frac{1}{4} = \sqrt{\frac{15}{16}}\frac{1}{4} + \sqrt{\frac{3}{4}}\frac{1}{4} + \sqrt{\frac{7}{16}}\frac{1}{4} + 0 \approx .6239 \dots$$

(b) Since the area of the circle of radius r is πr^2 , the area of a circle with radius 1 is $\pi 1^2 = \pi$. Thus, since the number in question is $\frac{1}{4}$ of that, the actual area is $\frac{\pi}{4} \approx .7853 \dots$

5. (a) $\int \sin x dx = -\cos x + C$, so

$$\int_0^\pi \sin x dx = (-\cos \pi) - (-\cos 0) = 1 + 1 = 2.$$

(b) $\int x^3 + x dx = \frac{1}{4}x^4 + \frac{1}{2}x^2 + C$. So,

$$\int_{-2}^2 x^3 + x dx = \left(\frac{1}{4}(2)^4 + \frac{1}{2}(2)^2\right) - \left(\frac{1}{4}(-2)^4 + \frac{1}{2}(-2)^2\right) = 0.$$