# Quiz 3 Solutions and Comments

By: A Tessellation of Hyperbolic Space

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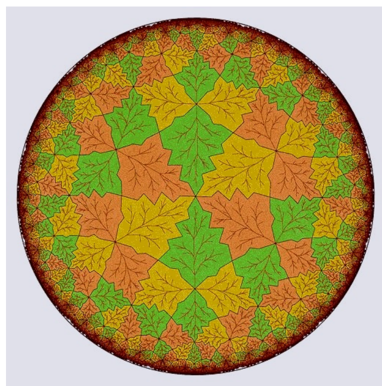

*Hi kids, I'm a wonderful picture of what hyperbolic space looks like. In case you're curious, it's a place where distances aren't what they seem. For instance, in my world, all of the leaves above are of the same size, making walking to the edge of the circle an impossible task. Here are the quiz #3 solutions!* ROCK ON!

Grade breakdown, remember everything is out of 50 points:

| Score | Number of people scoring that |
|---|---|
| $45 - \infty$ | 9 |
| $40 - 44$ | 2 |
| $35 - 39$ | 0 |
| $30 - 34$ | 2 |
| $-\infty - 29$ | 2 |

1. We use $u = \cos x$, and $du = -\sin x dx$. We save a $\sin x$ for later:

$$\int \sin^3 x \cos^4 x dx = \int \sin^2 x \cos^4 x \sin x dx = \int (1-\cos^2 x) \cos^4 x \sin x dx = -\int (1-u^2) u^4 du\,.$$

We distribute through the $u^4$ and continue:

$$-\int (1-u^2)u^4 du = \int u^6 - u^4 du = \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + C\,.$$

2. We use $u = \sec x, du = \sec x \tan x dx$. Then

$$\int \sec^3 x \tan x dx = \int \sec^2 x \sec x \tan x dx = \int u^2 du = \frac{1}{3}\sec^3 x + C\,.$$

3. We make the trig substitution $t = \sin\theta$, and so $dt = \cos\theta d\theta$. For later use, we will now find out what $\theta$ is if $t = 0$ and $\frac{\sqrt{3}}{2}$. These are the $\theta$ between 0 and $\pi/2$ where $\sin\theta = 0$ and $\sin\theta = \frac{\sqrt{3}}{2}$. So $\theta = 0$ and $\frac{\pi}{3}$, respectively. Now it is just a substitution game:

$$\begin{aligned}
\int_{t=0}^{\sqrt{3}/2} \tfrac{t^2}{(1-t^2)^{3/2}} dt &= \int_{\theta=0}^{\pi/3} \tfrac{\sin^2\theta}{(1-\sin^2\theta)^{3/2}} \cos\theta d\theta \\
&= \int_{\theta=0}^{\pi/3} \tfrac{\sin^2\theta}{(\cos^2\theta)^{3/2}} \cos\theta d\theta \\
&= \int_{\theta=0}^{\pi/3} \tfrac{\sin^2\theta}{\cos^2\theta} d\theta \\
&= \int_{\theta=0}^{\pi/3} \tan^2\theta d\theta \\
&= \int_{\theta=0}^{\pi/3} \sec^2\theta - 1 d\theta \\
&= \tan\theta - \theta|_{\theta=0}^{\pi/3} \\
&= \tan\tfrac{\pi}{3} - \tfrac{\pi}{3} = \sqrt{3} - \tfrac{\pi}{3}\,.
\end{aligned}$$

4. For this, one could use either $u$ substitution, trig substitution, or partial fractions. The solution I'll give is with partial fractions. Notice the roots of the denominator are $x = \pm 1$. So $x^2 - 1 = (x+1)(x-1)$. Now we know to expect:

$$\frac{3x}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}\,.$$

Clearing denominators gives $3x = A(x-1) + B(x+1) = (A+B)x + (B-A)$. Comparing coefficients gives $A + B = 3$, and $B - A = 0$. The second equation says $A = B$, and using that in the first equation gives $A = B = \frac{3}{2}$. So

$$\int \frac{3x}{x^2-1} dx = \int \frac{3/2}{x+1} + \frac{3/2}{x-1} dx = \frac{3}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + C\,.$$

## Rock on!!!!