

Math 212 Exam #1 solutions and comments

By: Stan, the friendly computer helper

October 20, 2006



Hi kids, I'm the friendly computer helper named Stan. I recently helped a helpless Corey set up his internet at home. So now, it's possible for Corey to check emails at home which is great. Maybe now he can buy some chains so he doesn't die in the wintertime driving down the mountain. It's really a shame he's so bad with computers. His incessant whining have made me very tired and have given me a headache. In fact, I don't feel at all like being amusing, and I just want to tell everyone the answers so I can get back to my cozy computer box and sleep for a number of hours that is less than 6. That's not really that bad, I guess I'm getting old. At any rate, I watched him grade the exams, and I wanted to give you all the solutions and a grade breakdown in hopes it will help you.

Score	Number of people who scored in that range
90 – ∞	4
80 – 89	4
70 – 79	6
60 – 69	2
$-\infty$ – 59	2

1. **Part (a), Blue Exam:** Using 4 boxes, we have $\Delta x = \frac{2-1}{4} = \frac{1}{4}$. The $x_i = 1 + \frac{i}{4}$. So the points

$$x_0 = 1, x_1 = 1.25, x_2 = 1.5, x_3 = 1.75, x_4 = 2. \text{ So,}$$

$$f(x_0) = 5, f(x_1) = 6.9375, f(x_2) = 9.25, f(x_3) = 11.938, f(x_4) = 15.$$

So the left approximation:

$$\begin{aligned} f(x_0)\frac{1}{4} + f(x_1)\frac{1}{4} + f(x_2)\frac{1}{4} + f(x_3)\frac{1}{4} &= \frac{1}{4}(5 + 6.9375 + 9.25 + 11.938) \\ &\approx 8.28. \end{aligned}$$

The right approximation:

$$\begin{aligned} f(x_1)\frac{1}{4} + f(x_2)\frac{1}{4} + f(x_3)\frac{1}{4} + f(x_4)\frac{1}{4} &= \frac{1}{4}(6.9375 + 9.25 + 11.938 + 15) \\ &\approx 10.781. \end{aligned}$$

- Part (a), Pink Exam:** Using 4 boxes, we have $\Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$. The $x_i = 1 + \frac{i}{2}$. So the points

$$x_0 = 1, x_1 = 1.5, x_2 = 2, x_3 = 2.5, x_4 = 3. \text{ So,}$$

$$f(x_0) = 7, f(x_1) = 10.25, f(x_2) = 14, f(x_3) = 18.25, f(x_4) = 23.$$

So the left approximation:

$$\begin{aligned} f(x_0)\frac{1}{2} + f(x_1)\frac{1}{2} + f(x_2)\frac{1}{2} + f(x_3)\frac{1}{2} &= \frac{1}{2}(7 + 10.25 + 14 + 18.25) \\ &\approx 24.75. \end{aligned}$$

The right approximation:

$$\begin{aligned} f(x_1)\frac{1}{2} + f(x_2)\frac{1}{2} + f(x_3)\frac{1}{2} + f(x_4)\frac{1}{2} &= \frac{1}{2}(10.25 + 14 + 18.25 + 23) \\ &\approx 32.75. \end{aligned}$$

- Part (b), Blue Exam** We have $\Delta x = \frac{2-1}{n} = \frac{1}{n}$. The $x_i = 1 + \frac{i}{n}$. $f(x_i) = 3(1 + \frac{i}{n})^2 + (1 + \frac{i}{n}) + 1 = 5 + \frac{7i}{n} + \frac{3i^2}{n^2}$. So

$$\begin{aligned} \lim_{n \rightarrow \infty} f(x_i)\Delta x &= \lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} [5 + \frac{7i}{n} + \frac{3i^2}{n^2}] \frac{1}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} [\frac{5}{n} + \frac{7i}{n^2} + \frac{3i^2}{n^3}] \\ &= \lim_{n \rightarrow \infty} 5 + \frac{7}{n^2} \frac{n(n+1)}{2} + \frac{3}{n^3} \frac{n(n+1)(2n+1)}{6} \\ &= 5 + 7/2 + 1 = 9.5. \end{aligned}$$

Part (b), Pink Exam We have $\Delta x = \frac{3-1}{n} = \frac{2}{n}$. The $x_i = 1 + \frac{2i}{n}$. $f(x_i) = (1 + \frac{2i}{n})^2 + 4(1 + \frac{2i}{n}) + 2 = 7 + \frac{12i}{n} + \frac{4i^2}{n^2}$. So

$$\begin{aligned} \lim_{n \rightarrow \infty} f(x_i) \Delta x &= \lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} [7 + \frac{12i}{n} + \frac{4i^2}{n^2}] \frac{2}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} [\frac{14}{n} + \frac{24i}{n^2} + \frac{8i^2}{n^3}] \\ &= \lim_{n \rightarrow \infty} 14 + \frac{24}{n^2} \frac{n(n+1)}{2} + \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6} \\ &= 14 + 12 + 16/6 = 28/3. \end{aligned}$$

As an inanimate computer, I have to say that several people seemed to get themselves confused with this process. It really is only an exercise in notation, and addition. If you missed this, study hard the solutions that I typed out, they will likely be helpful!

2. Both exams had the same question here. In fact, it's the same question as the quiz, so I will refer you to the quiz solutions.

3. Both exams had the same question 3 as well, here goes:

(a) $\int_{\pi}^{\ln x} \cos t dt = \sin t \Big|_{t=\pi}^{\ln x} = \sin(\ln x) - \sin \pi = \sin(\ln x)$. There are a few people that are under the impression that the integral of the cosine function is the negative sine function. This isn't the case: $\int \cos x dx = \sin x + C$!

(b) The derivative $\frac{d}{dx} [\sin(\ln x)] = \cos(\ln x) \cdot \frac{1}{x}$.

(c) Using the FTC, the derivative of F can be found by first plugging $(\ln x)$ in for t in the integrand $\cos t$, but since $\ln x$ is not x , we must multiply (according to the chain rule) by the derivative of $\ln x$, which is $\frac{1}{x}$. Corey tells me that you did a lot of these in class, be sure to check your notes!

4. **Blue exam:** (a) $\frac{1}{3}x^3 - 3x^2 + x + C$,

(b) $\sin x + \cos x + C$.

(c) For part (c), some people erroneously simplified the integrand. You should have gotten:

$$\frac{\sqrt{x} - 1}{\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}} = 1 - \frac{1}{\sqrt{x}}.$$

Now we integrate to get $x - 2\sqrt{x} + C$.

I should point out that several people made a common mistake whilst integrating. This mistake is to look at a product of functions, integrate each one separately, and multiply the result together. This is almost never how it goes, so if you found yourself expressing $\frac{\sqrt{x}-1}{\sqrt{x}} = (\sqrt{x} - 1)x^{-1/2}$, that's okay. But to integrate and get $(\frac{2}{3}x^{3/2} - x) \cdot 2x^{1/2}$, you've made that mistake.

(d) Let $u = x^3 + x =$ the entire denominator. Then $du = (3x^2 + 1)dx$, and the integral turns into, after substitution, $\int \frac{1}{u} du$. So the answer is $\ln|x^3 + x| + C$.

Pink exam: (a) $x^4 + \frac{3}{2}x^2 - 6x + C$,

(b) $-\cos x - \frac{2}{3}x^{3/2} + C$.

(c) For part (c), some people erroneously simplified the integrand. You should have gotten:

$$\frac{\sqrt{x} - 1}{\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}} = 1 - \frac{1}{\sqrt{x}}.$$

Now we integrate to get $x - 2\sqrt{x} + C$.

I should point out that several people made a common mistake whilst integrating. This mistake is to look at a product of functions, integrate each one separately, and multiply the result together. This is almost never how it goes, so if you found yourself expressing $\frac{\sqrt{x}-1}{\sqrt{x}} = (\sqrt{x}-1)x^{-1/2}$, that's okay. But to integrate and get $(\frac{2}{3}x^{3/2} - x) \cdot 2x^{1/2}$, you've made that mistake.

(d) Let $u = x^2 - x =$ the entire denominator. Then $du = (2x - 1)dx$, and the integral turns into, after substitution, $\int \frac{1}{u} du$. So the answer is $\ln|x^2 - x| + C$.

5. Both exams had the same questions here. For part (a):

$$\int_0^4 4x^3 - 2x dx = x^4 - x^2 \Big|_0^4 = (4^4 - 4^2) - (0^4 - 0^2) = 240.$$

(b) This is from the quiz, also. See the solutions there.

(c) $\int_0^{\frac{\pi}{2}} \cos x e^{\sin x} dx$. I'd let $u = \sin x$, so $du = \cos x dx$, and the integral turns into $\int_{x=0}^{\frac{\pi}{2}} e^u dx = e^{\sin x} \Big|_{x=0}^{\frac{\pi}{2}} = e^{\sin \frac{\pi}{2}} - e^{\sin 0} = e^1 - e^0 = e - 1$.

ROCK ON!!!!