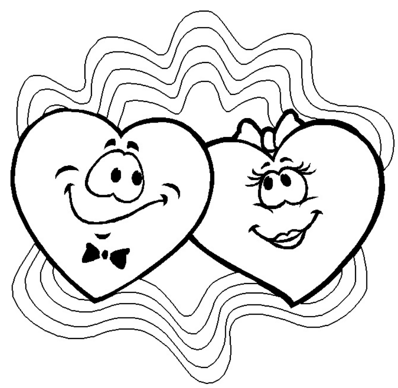
Exam Solutions

This pair of Valentine's Day Hearts

February 13, 2009



Hello kids! We're a pair of hearts that wishes you a happy Valentine's day! As a special gift, here are the solutions to the midterm. Enjoy!

1. Similarly to the quiz, the slopes are (a) 4, (b) 3.1, (c) 3.01, and these numbers seem to be heading towards (d) 3.

2. We claim the limit is 13. Let $\epsilon > 0$ be given. Then $|(6x-1)-13| = |6x-12| = 6|x-2| < 6\delta = \epsilon$. So set $\delta = \frac{\epsilon}{2}$.

3. (a) is the same question from the quiz.

   (b)
$$\lim_{x\to 5} \frac{x^2-9x+20}{x^2-5x} = \lim_{x\to 5} \frac{(x-5)(x-4)}{x(x-5)} = \lim_{x\to 5} \frac{x-4}{x} = \frac{1}{5}.$$

   (c)
$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)(x-3)}{x-3} = \lim_{x\to 3} x+3 = 6.$$

4. These answers can be seen from the graph, which is difficult to reproduce here. But be sure to see Corey if you need any clarification on these answers. (a) 2, (b) Does not exist, (c) 3, (d) 2, (e) No, there is not a removable discontinuity there (it is a non-removable discontinuity), (f) Yes.

5.

$$
\begin{aligned}
\lim_{\Delta x\to 0}\tfrac{f(x+\Delta x)-f(x)}{\Delta x} &= \lim_{\Delta x\to 0}\tfrac{2(x+\Delta x)^2-(x+\Delta x)-(2x^2-x)}{\Delta x}\\
&= \lim_{\Delta x\to 0}\tfrac{2(x^2+2x\Delta x+(\Delta x)^2)-x-\Delta x-2x^2+x}{\Delta x}\\
&= \lim_{\Delta x\to 0}\tfrac{2x^2+4x\Delta x+2(\Delta x)^2-x-\Delta x-2x^2+x}{\Delta x}\\
&= \lim_{\Delta x\to 0}\tfrac{4x\Delta x+2(\Delta x)^2-\Delta x}{\Delta x}\\
&= \lim_{\Delta x\to 0} 4x+2(\Delta x)-1\\
&= 4x-1.
\end{aligned}
$$

6. The derivative of the function is $f'(x) = 6x$:

$$
\begin{aligned}
\lim_{\Delta x\to 0}\tfrac{f(x+\Delta x)-f(x)}{\Delta x} &= \lim_{\Delta x\to 0}\tfrac{3(x+\Delta x)^2-3x^2}{\Delta x}\\
&= \lim_{\Delta x\to 0}\tfrac{3(x^2+2x\Delta x+(\Delta x)^2)-3x^2}{\Delta x}\\
&= \lim_{\Delta x\to 0}\tfrac{3x^2+6x\Delta x+3(\Delta x)^2-3x^2}{\Delta x}\\
&= \lim_{\Delta x\to 0}\tfrac{6x\Delta x+3(\Delta x)^2}{\Delta x}\\
&= \lim_{\Delta x\to 0} 6x+3\Delta x\\
&= 6x.
\end{aligned}
$$

And so now we can complete the problem. The slope of the tangent line of the curve at $x = 2$ is $f'(2) = 6(2) = 12$. The y-coordinate of the point in question is $f(2) = 3(2)^2 = 12$. So the equation of the tangent line, in point-slope form, is

$$y - 12 = 12(x - 2).$$