Quiz 1 Solutions and Comments

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NARF! Enjoy these solutions! I hope they help you study for the exam! ROCK ON!

1. The slopes are just the change in y divided by the change in x, so the answers are (a) (4-3)/(2-1) = 1, (b) (3.19-3)/(1.-1)=1.9, (c) (3.0199-3)/(1.01-1) = 1.99, and these seem to be heading towards (d) 2.

2. Let $\epsilon > 0$ be given. I must find a δ so that $|(2x+1)-13| < \epsilon$ whenever $0 < |x-6| < \delta$. Since $|(2x+1)-13| = |2x-12| = 2|x-6|$, we can choose $\delta = \epsilon/2$. Then $2|x-6| < 2\delta = \epsilon$.

   Pinky note: It's common that when I grade Calculus papers that have to do with this sort of problem that I see a lot of jumbled inequalities and some stuff that doesn't make any sense. Remember that this problem asks for a careful argument that the limit is what it is. So for those that missed points, take a look at the above and hopefully you'll see more about how it goes. But be sure to ask Corey in his office hours if you're still not sure.

3. For this problem, remember that *limits don't have anything at all to do with the actual output of the function at the point you're interested in.* My friend, The Brain, mutters that all the time as he walks around the lab, before bopping me on the head and telling me that we're going to take over the world. Most everyone who missed points on this one missed them because they wrote down the output of the function,

rather than the limit (should it exist at all). All the same, the answers are (a) 2, (b) DNE.

4. (a) $\lim\limits_{x\to 2} \frac{x^2-16}{x-4} = \lim\limits_{x\to 2} x + 4 = 6.$

   (b) $\lim\limits_{x\to 1} \sin\left(x^2 - 1 + \pi\right) = \sin\left(\lim\limits_{x\to 1} x^2 - 1 + \pi\right) = \sin \pi = 0\,.$

   Of course, what he actually typed out was $\lim\limits_{x\to 1} f(x) = \sin\left(x^2 - 1 + \pi\right)$. I think the problem was that he didn't consult me before printing the quiz up. When I saw the quiz earlier today and pointed out the error, Corey was livid. I had to set him up playing some violent video games just to calm him down. That's why I'm writing this, and he isn't.

   (c) For $f(x) = 2x^2$, we have

$$\begin{aligned}
\lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} &= \lim_{\Delta x\to 0} \frac{2(x+\Delta x)^2-2x^2}{\Delta x} \\
&= \lim_{\Delta x\to 0} \frac{2x^2+4x\Delta x+2(\Delta x)^2-2x^2}{\Delta x} \\
&= \lim_{\Delta x\to 0} 4x + 2\Delta x \\
&= 4x.
\end{aligned}$$

So, there you have it. Back to the lab to prepare for tomorrow night. One more thing, Corey told me not to forget to tell you:

Rock on!!!!