

Solutions to the Math 211 Midterm!!!

The state of Alaska

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Hi everyone! I'm the largest state in the USA, and I'm writing your exam solutions. Enjoy!!!!

- (a) The slope is $(7 - 3)/(3 - 2) = 4/1 = 4$. (b) $(3.31 - 3)/(2.1 - 2) = .31/.1 = 3.1$. (c) $(3.0301 - 3)/(2.01 - 2) = .0301/.01 = 3.01$. (d) These numbers seem to be approaching 3. (e) We compute the derivative of the function as $f'(x) = 2x - 1$, and at $x = 2$, we compute $f'(2) = 3$.
- Let $\epsilon > 0$ be given. Assume such a δ exists. Then $|(8x + 1) - 25| = |8x - 24| = 8|x - 3| < 8\delta$. So set $8\delta = \epsilon$, or $\delta = \epsilon/8$.
- (a) See the quiz solutions for this one.
(b) The numerator factors as $(x - 5)(x - 4)$, so

$$\lim_{x \rightarrow 5} \frac{x^2 - 9x + 20}{x^2 - 5x} = \lim_{x \rightarrow 5} \frac{(x - 5)(x - 4)}{x(x - 5)} = \lim_{x \rightarrow 5} \frac{x - 4}{x} = \frac{1}{5}$$

(c) $\lim_{x \rightarrow 2} \frac{x-2}{x^2-4} = \lim_{x \rightarrow 2} \frac{x-2}{(x+2)(x-2)} = \lim_{x \rightarrow 2} \frac{1}{x+2} = \frac{1}{4}$.

- (a) 1, (b) Does not exist, (c) 1.

5. For $f(x) = 2x^2 - x$, we have

$$\begin{aligned}\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} &= \lim_{\Delta x \rightarrow 0} \frac{2(x+\Delta x)^2 - (x+\Delta x) - (2x^2 - x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - x - \Delta x - 2x^2 + x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{4x\Delta x + 2(\Delta x)^2 - \Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 4x + 2\Delta x - 1 \\ &= 4x - 1.\end{aligned}$$

6. let $f(x) = 3x^2 + 1$. Then $f'(x) = 6x$, so $f'(2) = 12$. Notice also $f(2) = 13$, so the equation is $y - 13 = 12(x - 2)$, or $y = 12x - 11$.

7. (a) $f'(x) = 2x$.

(b) $f'(x) = 5 \cdot \frac{1}{2}x^{-1/2} + (-1)x^{-2} = \frac{5}{2}x^{-1/2} - x^{-2}$.

(c) Notice that $f(x) = \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}} = x^{1/2} - x^{-1/2}$. So $f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$.