Quiz 3 Solutions and Comments

By: The Voices in Corey's Head

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Hi kids! These are the voices in Corey's head coming to you to deliver these solutions to you. We've been influencing Corey for years to do bad things, but the other day we actually tried to influence him to do something good! He screwed one of the questions up and Corey realized this too late. It's question 2(a), see below for more of an explanation. So we decided to write these solutions out and go back into Corey's head. He never listens to us anyway. ROCK ON!

Grade breakdown, remember everything is out of 50 points:

Score	Number of people scoring that
$45 - \infty$	13
$40 - 44$	1
$35 - 39$	4
$30 - 34$	5
$-\infty - 29$	2

1. Let's call x : the distance from the base of the ladder to the house, and y : the height of the ladder on the house. Since the house is at a right angle, and the ladder is 25 feet long, we have the relationship $x^2 + y^2 = 25^2$. Some of you misplaced the 25 in this equation and inadvertantly were describing a situation where the house was angled and the ladder was straight up in the air. Back to the solution: $\frac{dx}{dt} = 2$, and we want to find $\frac{dy}{dt}$. So we differentiate:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0, \text{ so } \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}.$$

   When $x = 15$, $y = \sqrt{25^2 - 15^2} = 20$. So $\frac{dy}{dt} = \frac{-3}{2}$.

2. (a) This is the one Corey screwed up. Rolle's Theorem says that if a function is 1. continuous on $[a, b]$, 2. Differentiable on (a, b), and 3. $f(a) = f(b)$, then there exists a place c in (a, b) so that $f'(c) = 0$. The function I gave you satisfies 1 and 2 on

the interval given, but not 3. See, Corey originally typed this question out with the interval $[0, 2]$, and then everything would have worked. Although he also knew he'd be asking you question (c) about the global min and max, and, at the last minute, altered the interval so there was only one place for a global max, at $x = 3$. He didn't realize this screwed up part (a), and as a result gave everyone full credit for it. We tried to tell him.

(b) The critical points are when $f'(x) = 2 - 2x = 0$, and the places where $f'(x)$ does not exist. Since the function is defined on an interval, we must include the endpoints as critical points as well. So the answer is $x = 0, 1, 3$.

(c) Testing each critical point, the global max is at $x = 1$ and the global min is at $x = 3$.

3. If $f(x) = 3x^5 - 5x^3$, then $f'(x) = 15x^4 - 15x^2 = 15x^2(x^2 - 1)$. So the critical points are $x = \pm 1, 0$. Just because a point is a critical point doesn't mean the function has local extrema there! This is the point of the first derivative test. On the number line, I get:

$$\nearrow \quad \searrow \quad \searrow \quad \nearrow$$

$$-1 \qquad 0 \qquad 1$$

So the first derivative test says there is a local max at $x = -1$, and a local min at $x = 1$.

ROCK ON!!!!