Math 211 Midterm #1 Solutions and Comments

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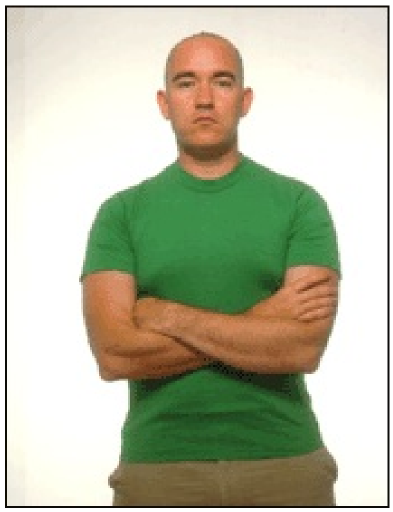

Hi folks, I'm here to help you clean your floors and share with you some of Corey's thoughts about the exam, and the solutions. I've peeked at the grade breakdown whilst pretending to clean Corey's office, and I share with you below what I found. Overall, the exams were very good, the average was about 83. Below are the solutions to the problems on the exam, along with some of my thoughts.

Score Range	# of people within that range
90-∞	14
80-89	5
70-79	4
60-69	3
$-\infty$ - 59	3

1. Both exams had the same problem to do. The answers for parts (a) - (d) are 4, 3.1, 3.01, and 3. Of course, if you had a reasonable guess that your numbers seemed to

go towards, you could have gotten something other than 3 and still got full credit. For part (e), I'll share with you how I think it goes:

$$\begin{array}{lrcl}
\text{First:} & f(2+\Delta x) & = & (2+\Delta x)^2-(2+\Delta x)+1 \\
 & & = & 4+4\Delta x+(\Delta x)^2-2-\Delta x+1 \\
\text{Second:} & f(2+\Delta x)-f(2) & = & 4+4\Delta x+(\Delta x)^2-2-\Delta x+1-3 \\
 & & = & 4\Delta x+(\Delta x)^2-\Delta x \\
\text{Third:} & \frac{f(2+\Delta x)-f(2)}{\Delta x} & = & 4+\Delta x-1 \\
\text{Last:} & \lim\limits_{\Delta x\to 0}\frac{f(2+\Delta x)-f(2)}{\Delta x} & = & 4-1=3\,.
\end{array}$$

There were several people who did not do this calculation correctly. Remember, in class, Corey mentioned several mistakes common to this process. See my comments on problem 5 if you made a mistake and still don't know why. I'll save the almost identical discussion for there.

2. **Yellow exam**: Let $\epsilon > 0$ be given. Then I must show there exists a δ so that $|(8x+1)-25| < \epsilon$ whenever $0 < |x-3| < \delta$. But

$$|(8x+1)-25| = |8x-24| = 8|x-3| < 8\delta\,.$$

So set $\delta = \epsilon/8$.

Green exam: Let $\epsilon > 0$ be given. Then I must show there exists a δ so that $|(5x+17)-22| < \epsilon$ whenever $0 < |x-1| < \delta$. But

$$|(5x+17)-22| = |5x-5| = 5|x-1| < 5\delta\,.$$

So set $\delta = \epsilon/5$.

As I was rifling through the exams I noticed that although a lot of people got the right δ, a lot of others did not. Also, a lot of those who got the right δ sort of messed up the argument. Corey seems like a pretty reasonable guy, and it doesn't look like he took off many points unless he had to, but all the same, I suggest that you see how the above proof differs from your answer, and determine how, if at all, the solutions differ.

3. **Yellow exam**:

(a) $\lim\limits_{x\to 1}[\sin\left(x^2-1+\pi\right)] = \sin\pi = 0$.

(b) $\lim\limits_{x\to 2^+}\frac{|x-2|}{x-2} = 1$.

(c) $\lim\limits_{x\to-5}\left[\frac{x^2+3x-10}{x^2+5x}\right] = \lim\limits_{x\to-5}\left[\frac{(x+5)(x-2)}{x(x+5)}\right] = \lim\limits_{x\to-5}\left[\frac{(x-2)}{x}\right] = \frac{-7}{-5} = 7/5.$

(d) $\lim\limits_{x\to-1}\left[\frac{x^2}{x^2-1}\right]$ Does Not Exist.

(e) $\lim\limits_{x\to7^-}[\frac{1}{x-7}] = -\infty.$

Green exam:

(a) $\lim\limits_{x\to-2}\left[\frac{x^2+x-2}{x^2+2x}\right] = \lim\limits_{x\to-2}\left[\frac{(x+2)(x-1)}{x(x+2)}\right] = \lim\limits_{x\to-2}\left[\frac{(x-1)}{x}\right] = \frac{-3}{-2} = 3/2.$

(b) $\lim\limits_{x\to3}[\frac{1}{x-3}]$ Does Not Exist.

(c) $\lim\limits_{x\to2^-}\frac{|x-2|}{x-2} = -1$

(d) $\lim\limits_{x\to1}[\sin\left(x^2-1+\pi\right)] = \sin\pi = 0$

(e) $\lim\limits_{x\to2}\left[\frac{x^2+1}{x^2-4}\right]$ Does Not Exist.

I think, for those unsure of how I got these answers, that you should look at some of the subtleties that are written in the problems. For instance, in part (b) of the yellow and part (c) of the green exam, there is a + of − in the superscript of the 2. I think some of you may have brushed over that. Another thing I noticed was that there are still a lot of people that think that $\frac{0}{0} = 0$, and that just because there is a limit of 0 in the denominator that means that a limit must either not exist or equal $\pm\infty$. Let me suggest that you look over your notes and the homework to help you see what some of the differences are.

4. **Yellow exam**: ∞, 2, -2, DNE, -3. **Green Exam**: 1, DNE, ∞, 1, 1. The graphs, of course, are different and difficult to include here. Again, look at your notes to see examples of determining limits in this manner.

5. **Yellow exam**: Consider the function $f(x) = 2x^2 - 7x + 1$.

$$\begin{array}{lrcl}
\text{First:} & f(x+\Delta x) & = & 2(x+\Delta x)^2 - 7(x+\Delta x) + 1 \\
 & & = & 2x^2 + 4x\Delta x + 2(\Delta x)^2 - 7x - 7\Delta x + 1 \\
\text{Second:} & f(x+\Delta x) - f(x) & = & 2x^2 + 4x\Delta x + 2(\Delta x)^2 - 7x - 7\Delta x + 1 - (2x^2 - 7x + 1) \\
 & & = & 4x\Delta x + 2(\Delta x)^2 - 7\Delta x \\
\text{Third:} & \frac{f(x+\Delta x)-f(x)}{\Delta x} & = & 4x + 2\Delta x - 7 \\
\text{Last:} & \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} & = & 4x - 7\,.
\end{array}$$

Green exam: Consider the function $f(x) = 3x^2 + 4x - 2$.

$$\begin{array}{lrcl}
\text{First:} & f(x+\Delta x) & = & 3(x+\Delta x)^2 + 4(x+\Delta x) - 2 \\
 & & = & 3x^2 + 6x\Delta x + 3(\Delta x)^2 + 4x + 4\Delta x - 2 \\
\text{Second:} & f(x+\Delta x) - f(x) & = & 3x^2 + 6x\Delta x + 3(\Delta x)^2 + 4x + 4\Delta x - 2 - (3x^2 + 4x - 2) \\
 & & = & 6x\Delta x + 3(\Delta x)^2 + 4\Delta x \\
\text{Third:} & \frac{f(x+\Delta x)-f(x)}{\Delta x} & = & 6x + 3(\Delta x) + 4 \\
\text{Last:} & \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} & = & 6x + 4\,.
\end{array}$$

So, how about those comments I promised. I would say that the mistakes I saw on this problem and number 1(e) fell into two categories. The first kind of mistake was minor... an innocent misplaced minus sign, or a failure to distribute a number throughout the parentheses, etc. Those I usually saw in the second step. The second kind of mistake was more major. Remember, these were the mistakes I warned you about! Several of you made the mistake of expressing

$$f(x+\Delta x) = f(x) + \Delta x \longleftarrow \text{ This is almost never true!}$$

Consequently, a lot of you got the answer of 1 for your difference quotient. Think for a minute, though. Could this really be the case? If the above is always true, *regardless of the function*, then the slope of every tangent line of any function at any point would be 1. As our examples indicate, and question 1 was meant to show you, this is not always the case. Look very carefully through Corey's examples and the homework , and work hard to understand two things about this kind of problem. First, there is a lot of algebra involved that is almost always unavoidable. Second, there is no trick or shortcut. Or, for that matter, any choices. The difference quotient is just a long complicated list of directions that you must follow. If you understand the notation, then you understand the directions. So focus your efforts first on the notation.

Now that I've said all of that, I hope it helps. And remember,

ROCK ON!!!!