211 Midterm #2 Solutions

By: A disturbing Easter bunny and white dog

November 14, 2006

Hi kids! This is the Easter bunny. I've come from happy land where I live to deliver these solutions to you. Corey seems to be very busy lately for no apparent reason and hasn't been able to post these solutions. He seems to be spending a lot of time in his shame closet–I can't tell if it's because he hasn't posted these yet or not. Either way, since Easter is still a little while away I had some spare time and thought I would help him out. The grade breakdown is below. My little dog is telling you all to ROCK ON!

Score	Number of People who Scored that
90-∞	17
80-89	5
70-79	4
60-69	1
$-\infty$ - 59	2

1. Yellow Exam. To find the equation of a line one must have (among other things) a point on the line, and the slope of the line. The point is given to you, and so we differentiate to find the slope.

$$\frac{d}{dx}\left[\frac{x}{3x+2}\right] = \frac{(3x+2)\frac{d}{dx}[x] - x\frac{d}{dx}[3x+2]}{(3x+2)^2} = \frac{(3x+2)-3x}{(3x+2)^2} = \frac{2}{(3x+2)^2}.$$

Plugging in $x = 1$, we get $f'(1) = \frac{2}{25}$. Thus the equation of the line is

$$y - \frac{1}{5} = \frac{2}{25}(x-1), \text{ or } y = \frac{2}{25}x + \frac{3}{25}\,.$$

Green Exam. To find the equation of a line one must have (among other things) a point on the line, and the slope of the line. The point is given to you, and so we differentiate to find the slope.

$$\frac{d}{dx}\left[\frac{3x}{2x-1}\right] = \frac{(2x-1)\frac{d}{dx}[3x] - 3x\frac{d}{dx}[2x-1]}{(2x-1)^2} = \frac{(2x-1)3 - 3x\cdot 2}{(2x-1)^2} = \frac{-3}{(2x-1)^2}\,.$$

Plugging in $x = 1$, we get $f'(1) = -3$. Thus the equation of the line is

$$y - 3 = (-3)(x-1) \text{ or } y = -3x + 6\,.$$

2. Yellow Exam. Here are the answers... many of you may want to see some more details, if so, feel free to stop by Corey's office hours (or, by appointment as well) and discuss with him your misunderstanding. Corey told me that he indicated to people where their mistakes were on their exams to point them in the right direction. If those indications and these answers aren't enough for you to figure out where you went wrong, then be sure to contact Corey and he'll point you in the right direction.

   (a) $\frac{d}{dx}[x^4 + \sin x - 7x] = 4x^3 + \cos x - 7$.
   (b) $\frac{d}{dx}\left[\frac{5}{\sqrt{x}} - 2\sqrt{x}\right] = \frac{-5}{2}x^{-3/2} - x^{-1/2}$.
   (c) $\frac{d}{dx}\left[x\cos x - \sin x \cos x\right] = -x\sin x + \cos x + \sin^2 x - \cos^2 x$.
   (d) $\frac{d}{dx}\left[\frac{\cos x}{x^2+1}\right] = \frac{-(x^2+1)\sin x - \cos x(2x)}{(x^2+1)^2}$.
   (e) $\frac{d}{dx}\left[\sin(2x + x^2)\right] = \cos(2x+x^2)\cdot(2+2x)$.
   (f) $\frac{d}{dx}\left[\frac{x}{\cos(x^2+1)}\right] = \frac{\cos(x^2+1) + x\sin(x^2+1)\cdot(2x)}{\cos^2(x^2+1)}$.
   (g) For this one, we implicitly differentiate. So we differentiate both sides, collect all of the $\frac{dy}{dx}$ terms on one side and divide out:

$$\begin{aligned}
\frac{d}{dx}\left[\cos(y)\right] &= \frac{d}{dx}x^2y - 2\sin(y) \\
-\sin y\frac{dy}{dx} &= x^2\frac{dy}{dx} + 2xy - 2\cos(y)\frac{dy}{dx} \\
-\sin y\frac{dy}{dx} - x^2\frac{dy}{dx} + 2\cos y\frac{dy}{dx} &= 2xy \\
(-\sin y - x^2 + 2\cos y)\frac{dy}{dx} &= 2xy \\
\frac{dy}{dx} &= \frac{2xy}{-\sin y - x^2 + 2\cos y}\,.
\end{aligned}$$

Green Exam. Here are the answers... many of you may want to see some more details, if so, feel free to stop by Corey's office hours (or, by appointment as well) and discuss with him your misunderstanding. Corey told me that he indicated to people where their mistakes were on their exams to point them in the right direction. If those indications and these answers aren't enough for you to figure out where you went wrong, then be sure to contact Corey and he'll point you in the right direction.

(a) $\frac{d}{dx}[x^2 + \cos x - 5] = 2x - \sin x$.

(b) $\frac{d}{dx}\left[\frac{2}{\sqrt{x}} - 3\sqrt{x}\right] = -x^{-3/2} - \frac{3}{2}x^{-1/2}$.

(c) $\frac{d}{dx}\left[x^2 \cos x - \sin^2 x\right] = -x^2 \sin x + 2x \cos x - 2 \sin x \cos x$.

(d) $\frac{d}{dx}\left[\frac{\cos x}{x+3x^2}\right] = \frac{-(x+3x^2)\sin x - \cos x(1+6x)}{(x+3x^2)^2}$.

(e) $\frac{d}{dx}\left[\sin(2x + x^2)\right] = \cos(2x + x^2) \cdot (2 + 2x)$.

(f) $\frac{d}{dx}\left[\frac{x^{14}}{\cos(x+1)}\right] = \frac{\cos(x+1) \cdot 14x^{13} + x^{14}\sin(x+1)}{\cos^2(x+1)}$.

(g) For this one, we implicitly differentiate. So we differentiate both sides, collect all of the $\frac{dy}{dx}$ terms on one side and divide out:

$$\begin{aligned}
\frac{d}{dx}\left[\cos(y)\right] &= \frac{d}{dx}x^2y - 2\sin(y) \\
-\sin y\frac{dy}{dx} &= x^2\frac{dy}{dx} + 2xy - 2\cos(y)\frac{dy}{dx} \\
-\sin y\frac{dy}{dx} - x^2\frac{dy}{dx} + 2\cos y\frac{dy}{dx} &= 2xy \\
(-\sin y - x^2 + 2\cos y)\frac{dy}{dx} &= 2xy \\
\frac{dy}{dx} &= \frac{2xy}{-\sin y - x^2 + 2\cos y}.
\end{aligned}$$

3. Yellow Exam. We implicitly differentiate to get:

$$\begin{aligned}
\frac{d}{dx}[x^2 + y^2] &= \frac{d}{dx}[25] \\
2x + 2y\frac{dy}{dx} &= 0. \\
\text{So, at } (3,4): & \\
2 \cdot 3 + 2 \cdot 4 \cdot \frac{dy}{dx} &= 0 \\
\text{So } \frac{dy}{dx} &= -\frac{3}{4}.
\end{aligned}$$

Yellow Exam. We implicitly differentiate to get:

$$\begin{aligned}
\frac{d}{dx}[x^2 + y^2] &= \frac{d}{dx}[169] \\
2x + 2y\frac{dy}{dx} &= 0. \\
\text{So, at } (5, 12): & \\
2 \cdot 5 + 2 \cdot 12 \cdot \frac{dy}{dx} &= 0 \\
\text{So } \frac{dy}{dx} &= -\frac{5}{12}.
\end{aligned}$$

4. For $y = x + \frac{1}{x}$, we have

$$\begin{aligned}
y' = \frac{dy}{dx} &= 1 - \frac{1}{x^2} \\
y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} &= \frac{d}{dx}[1 - x^{-2}] \\
&= 2x^{-3} \\
&= \frac{2}{x^3}.
\end{aligned}$$

Well, that's all from Happy Land, I hope these solutions help everyone to be good boys and girls! My little dog is again telling me and all of you to:

ROCK ON!!!