Midterm # 1 B Solutions

The BCS System

November 12, 2009

Hi everyone! Here are the solutions to the Math 110 Exam of Happiness B!!! Enjoy!

1. (a) $f(-2) = \frac{(-2)^2+4}{-2} = \frac{8}{-2} = 4$.

   (b) There are no y-intercepts, since 0 is not in the domain of the function.

   (c) The function is odd, since

   $$f(-x) = \frac{(-x)^2+4}{-x} = -\frac{x^2+4}{x} = -f(x).$$

2. (a) Shift the graph 3 units to the right.

   (b) Reflect the graph over the x-axis, and scale the graph away the x-axis by a factor of 3.

   (c) Scale horizontally towards the y-axis by a factor of 3, and then shift the graph down 6 units.

3. The midpoint between the points P and Q is

   $$\left(\frac{5+(-1)}{2}, \frac{2+(-6)}{2}\right) = (2, -2).$$

4. We complete the square on the x's to see that

   $$x^2 + 4x = x^2 + 4x + 4 - 4 = (x+2)^2 - 4.$$

   Completing the square with the y's gives us

   $$y^2 - 12y = y^2 - 12y + 36 - 36 = (y-6)^2 - 36.$$

   So finally we have

   $$9 = x^2 + 4x + y^2 - 12y = (x+2)^2 - 4 + (y-6)^2 - 36,$$

   So,

   $$49 = (x+2)^2 + (y-6)^2.$$

   Thus, the center of the circle is at $(-2, 6)$ and the radius is $\sqrt{49} = 7$.

5. This line has slope -6, and goes through $(5, 7)$, so the equation is

$$y - 7 = -6(x - 5).$$

6. This line has slope $\frac{-1}{7}$, and goes through $(2, -1)$, so the equation is

$$y + 1 = -\frac{1}{7}(x - 2).$$

7. (a) $(f - g)(1) = f(1) + g(1) = [(1) - (1)^2] - [(2(1) + 1] = -3$.

   (b) The rule would be

$$(f \cdot g)(x) = f(x)g(x) = (x - x^2)(2x + 1).$$

   (c) The rule would be

$$(g \circ f)(x) = g(f(x)) = g(x - x^2) = 2(x - x^2) + 1.$$

8. (a) We notice that

$$-2x^2-16x = -2(x^2+8x) = -2(x^2+8x+16-16) = -2[(x+4)^2-16] = -2(x+4)^2+32.$$

   So

$$-2x^2 - 16x - 30 = -2(x + 4)^2 + 32 - 30 = -2(x + 4)^2 + 2$$

   (b) The vertex is at $(-4, 2)$, and the graph is pictured below.