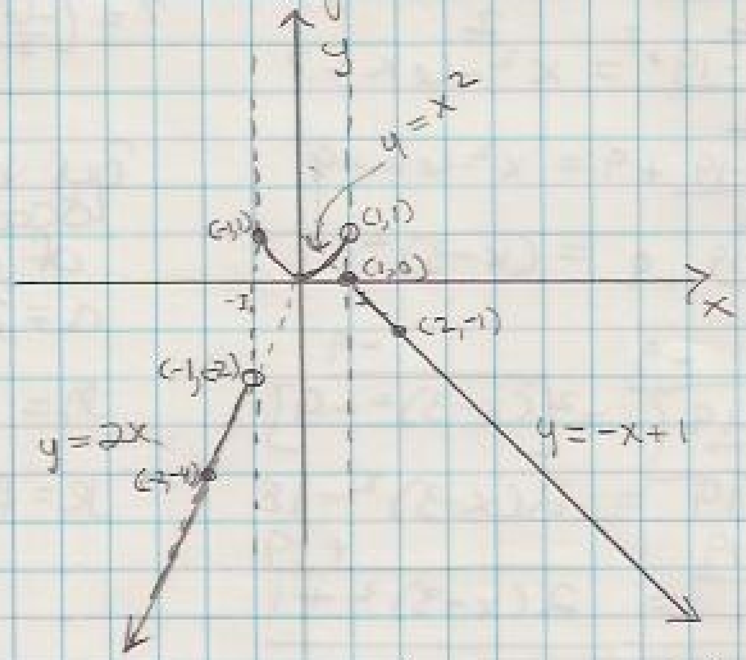
15. Sketch the piece-wise function defined as follows

$$f(x) = \begin{cases} 2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ -x+1 & \text{if } x \ge 1 \end{cases}$$



16. Express $f(x)$ in the form $f(x) = a(x-h)^2 + k$

$f(x) = -x^2 - 6x - 5$

Using complete the square

$$f(x) = -x^2 - 6x - 5$$
$$\underline{+5 \qquad\qquad +5}$$
$$\frac{f(x)+5}{-1} = \frac{-x^2-6x}{-1}$$
$$-f(x) - 5 \stackrel{+?}{=} x^2 + 6x + ? \qquad ? = \left(\frac{6}{2}\right)^2 = 3^2 = 9$$
$$-f(x) - 5 + 9 = x^2 + 6x + 9$$
$$-f(x) + 4 = (x+3)^2$$
$$\underline{-4 \qquad\qquad -4}$$
$$\frac{-f(x)}{-1} = \frac{(x+3)^2 - 4}{-1}$$
$$f(x) = -(x+3)^2 + 4$$

By theorem for locating vertex of a parabola.

$a = -1,\ b = -6,\ c = -5$

$$h = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3$$
$$k = f(-3) = -(-3)^2 - 6(-3) - 5$$
$$= -9 + 18 - 5$$
$$= 9 - 5$$
$$= 4$$

So $f(x) = a(x-h)^2 + k$

$= -1(x-(-3))^2 + 4$

$= -(x+3)^2 + 4$ ✓

Same instructions as #16

17. $f(x) = 2x^2 - 12x + 19$

By the complete the square method

$$\frac{-19 \qquad -19}{f(x) - 19 = 2x^2 - 12x}$$

$$\frac{f(x)-19}{2} = \frac{2x^2-12x}{2}$$

$$\frac{f(x)-19}{2} + ? = x^2 - 6x + ?$$

$? = \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$$\frac{f(x)-19}{2} + 9 = x^2 - 6x + 9$$

$$\frac{f(x)-19}{2} + 9 = (x-3)^2$$

$$-9 \qquad -9$$

$$2\left[\frac{f(x)-19}{2}\right] = 2\left[(x-3)^2 - 9\right]$$

$$f(x) - 19 = 2(x-3)^2 - 18$$

$$+19 \qquad +19$$

$$\underline{\underline{f(x) = 2(x-3)^2 + 1}}$$

By the theorem for locating the vertex of a parabola

$a = 2,\ b = -12,\ c = 19$

$h = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$

$k = f(3) = 2(3)^2 - 12(3) + 19$
$= 2(9) - 36 + 19$
$= 18 - 36 + 19$
$= -18 + 19$
$= 1$

so $f(x) = a(x-h)^2 + k$
$\underline{\underline{= 2(x-3)^2 + 1}}$ ✓

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18. (a) Use the quadratic formula to find the zeros of f.

(b) Find the maximum or minimum value of f(x)

(c) Sketch graph of f.

$$f(x) = 6x^2 + 7x - 24$$

(a) $x = \dfrac{-7 \pm \sqrt{7^2 - 4(6)(-24)}}{2(6)}$

$= \dfrac{-7 \pm \sqrt{49 + 576}}{12}$

$= \dfrac{-7 \pm \sqrt{625}}{12}$

$= \dfrac{-7 \pm 25}{12}$

$= \dfrac{-7 + 25}{12}$ and $\dfrac{-7 - 25}{12}$

$= \dfrac{18 \div 6}{12 \div 6}$ $\quad = \dfrac{-32 \div 4}{12 \div 4}$

$= \dfrac{3}{2}$ $\quad = \dfrac{-8}{3}$

So the zeros are at

$x = \{ \frac{3}{2}, -\frac{8}{3} \}$

And x-intercepts are

$(3/2, 0)$ and $(-8/3, 0)$

$= (1.5, 0)$ $\quad \approx (-2.67, 0)$

(b) since $a = 6 > 0$ then $f(-\frac{b}{2a})$ is the minimum value of f(x).

$a = 6,\ b = 7,\ c = -24$

$h = \dfrac{-b}{2a} = \dfrac{-7}{2(6)} = \dfrac{-7}{12}$

≈ -0.583

$f(-\frac{7}{12}) = 6\left(\frac{-7}{12}\right)^2 + 7\left(\frac{-7}{12}\right) - 24$

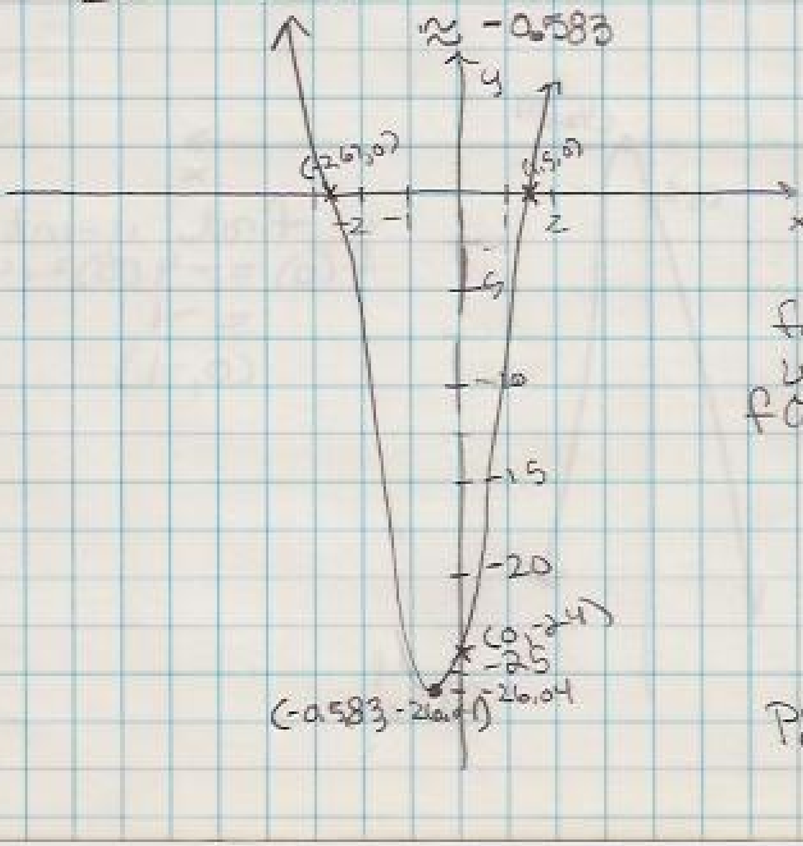
$= \frac{\cancel{6}}{1}\left(\frac{49}{\cancel{144}\,24}\right) - \frac{49}{12} - 24$

$= \frac{49}{24} - \frac{49}{12} - 24$

$= \frac{49}{24} - \frac{98}{24} - \frac{576}{24}$

$= \frac{-625}{24} \approx \underline{\underline{-26.04}}$ ← min of f(x)

(c)



find y-int let $x = 0$

$f(0) = 6(0)^2 + 7(0) - 24$

$= -24$

$(0, -24)$

19. $f(x) = -4x^2 + 4x - 1$

$a = -4,\ b = 4,\ c = -1$

(a) $x = \dfrac{-4 \pm \sqrt{4^2 - 4(-4)(-1)}}{2(-4)}$

$= \dfrac{-4 \pm \sqrt{16 - 16}}{-8}$

$= \dfrac{-4 \pm \sqrt{0}}{-8}$

$= \dfrac{-4}{-8}$

$= \dfrac{1}{2}$

so $\frac{1}{2}$ (mult. 2) is the zero of f.

(b) since $a = -4 < 0$ $f\left(-\frac{b}{2a}\right)$ is the maximum value of $f(x)$

$h = -\dfrac{b}{2a} = \dfrac{-4}{2(-4)} = \dfrac{4}{8} = \dfrac{1}{2}$

$k = f\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 1$

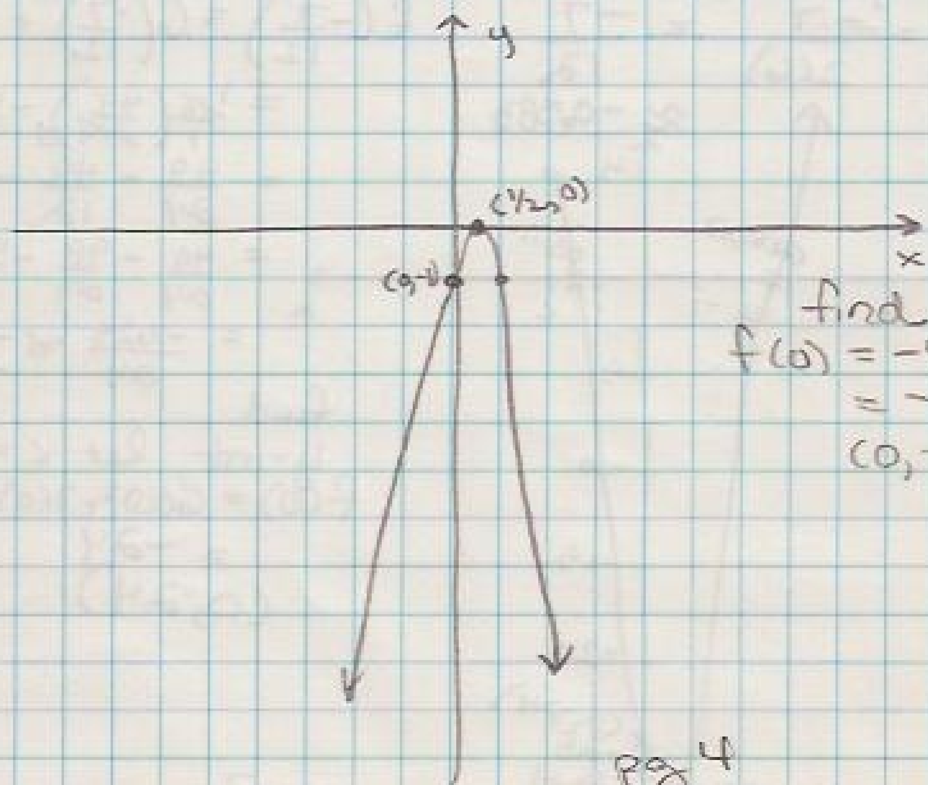
$= -4\left(\frac{1}{4}\right) + 2 - 1$

$= -1 + 2 - 1$

$= 0$ ← maximum value of $f(x)$

Vertex at $\left(\frac{1}{2}, 0\right)$

(c)



find y-int.

$f(0) = -4(0)^2 + 4(0) - 1$

$= -1$

$(0, -1)$

20. $f(x) = 2x^2 - 4x - 11$

$a = 2,\ b = -4,\ c = -11$

(a) $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-11)}}{2(2)}$

$= \dfrac{4 \pm \sqrt{16 + 88}}{4}$

$= \dfrac{4 \pm \sqrt{104}}{4}$

$\approx \dfrac{4 \pm 10.2}{4}$

$= \dfrac{4 + 10.2}{4}$ and $\dfrac{4 - 10.2}{4}$

$= 3.55$ $\qquad = -1.55$

So the zeros are $x = \underline{\{3.55, -1.55\}}$

and x-int. are $(3.55, 0)$ and $(-1.55, 0)$

(b) Since $a = 2 > 0$ $f\left(-\frac{b}{2a}\right)$ is the minimum value of $f(x)$

$h = \dfrac{-b}{2a} = \dfrac{-(-4)}{2(2)} = \dfrac{4}{4} = 1$

$k = f(1) = 2(1)^2 - 4(1) - 11 = 2 - 4 - 11 = -2 - 11 = -13$

So $\underline{-13}$ is the minimum value of $f(x)$.

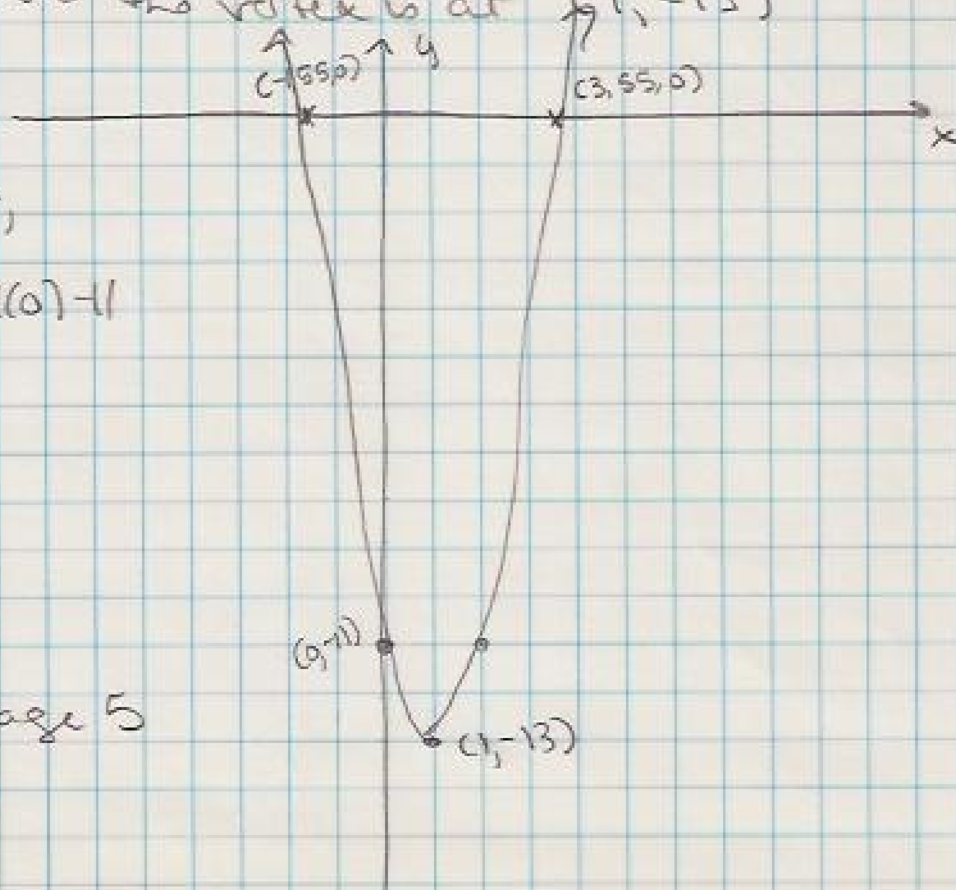
and the vertex is at $(1, -13)$

(c)

find y-intercept,
let $x = 0$

$f(0) = 2(0)^2 - 4(0) - 11$
$= -11$

$(0, -11)$



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