MIDTERM 1 REVIEW
PRACTICE PROBLEMS
SOLUTIONS
OCT. 28, 2009

1. Use remainder Thm. to find $f(-3)$ if $f(x) = x^3 + 2x^2 + 3x + 4$. Check your answer.

$$\begin{array}{r|rrrr} -3 & 1 & 2 & 3 & 4 \\ & & -3 & 3 & -18 \\ \hline & 1 & -1 & 6 & \boxed{-14} \end{array}$$

Our remainder is -14 so By remainder theorem $\underline{\underline{f(-3) = -14}}$

Check: $f(-3) = (-3)^3 + 2(-3)^2 + 3(-3) + 4$
$= -27 + 2(9) - 9 + 4$
$= -27 + 18 - 9 + 4$
$= -9 - 9 + 4$
$= -18 + 4$
$= -14$ ✓

2. Find a polynomial $f(x)$ with zeros at $0, -2, 3$ which satisfies $f(1) = 30$.

$f(x) = a(x-0)(x-(-2))(x-3)$
$f(x) = ax(x+2)(x-3)$

To find a, use $f(1) = 30$

$f(1) = a(1)(1+2)(1-3) = 30$
$a(3)(-2) = 30$
$\dfrac{-6a}{-6} = \dfrac{30}{-6}$
$a = -5$

so $f(x) = -5x(x+2)(x-3)$
$= -5x(x^2 - 3x + 2x - 6)$
$= -5x(x^2 - x - 6)$
$= \underline{\underline{-5x^3 + 5x^2 + 30x}}$

3. Show that 2 is a zero of $f(x)$ of mult. 2 and express $f(x)$ as a product of linear factors.

$f(x) = 3x^4 - 5x^3 - 12x^2 + 12x + 16$

$$\begin{array}{r|rrrrr} 2 & 3 & -5 & -12 & +12 & +16 \\ & & 6 & 2 & -20 & -16 \\ \hline 2 & 3 & 1 & -10 & -8 & \boxed{0} \\ & & 6 & 14 & 8 & \\ \hline & 3 & 7 & 4 & \boxed{0} & \end{array}$$

So 2 is a zero of at least mult. 1.

so now we know 2 is a zero of mult. 2.

$f(x) = (x-2)^2(3x^2 + 7x + 4)$
$3x^2 + 3x + 4x + 4$
$3x(x+1) + 4(x+1)$

$f(x) = \underline{(x-2)^2(3x+4)(x+1)}$

4. Use Descartes' Rule of Signs to determine the # of possible positive, negative real and nonreal complex solutions to

$$f(x) = x^5 - 3x^4 + 2x^3 - 6x + 3$$

\# possible positive real solutions

= # of variations in sign of $f(x)$ or is less than that # by an even integer.

so $f(x) = \underbrace{x^5 - 3x^4}_{1}\underbrace{\;+2x^3}_{2,\,3}\underbrace{-6x + 3}_{4}$

has 4 variations in sign, so there are 4, 2, or 0 positive real solutions

\# negative real solutions = the # of variations in sign of $f(-x)$

so $f(-x) = (-x)^5 - 3(-x)^4 + 2(-x)^3 - 6(-x) + 3$

$= -x^5 - 3x^4 \underbrace{- 2x^3 + 6x}_{1} + 3$

has 1 variation in sign so there is 1 negative real solution

To find nonreal complex solutions we need the table.

# positive real solutions	4	2	0	
# negative real solutions	1	1	1	
# nonreal complex solutions	0	2	4	
Total	5	5	5	← equal to degree of $f(x)$

5. Find a polynomial w/ real coefficients and leading coeff. 1 having the given zeros. Express $f(x)$ as a product of linear and quadratic polynomials with real coefficients that are irreducible over the real numbers.

Zeros are $3, 0, 2+i$; degree 4

since $2+i$ is a zero, so is $2-i$

So

$f(x) = 1(x-3)(x-0)(x-(2+i))(x-(2-i))$

$= (x-3)x(x-2-i)(x-2+i)$ ← multiply these

$$\underline{f(x) = x(x-3)(x^2-4x+5)}$$

	x	-2	$+i$
x	x^2	$-2x$	$\cancel{+ix}$
-2	$-2x$	$+4$	$\cancel{-2i}$
$-i$	$\cancel{-ix}$	$\cancel{+2i}$	$-i^2 = -(-1) = +1$

$= x^2 - 4x + 5$

6. Use Theorem on Rational Zeros of a polynomial to find all solutions to

$$f(x) = x^4 + 3x^3 - 12x^2 - 6x + 20 = 0$$

possible solutions are in form c/d with c being factors of 20 and d factors of 1

$$c/d = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$$

Now find solutions by trial and error.

1	1	3	−12	−6	+20
		1	4	−8	−14
	1	4	−8	−14	6

−1	1	3	−12	−6	+20
		−1	−2	14	−8
	1	2	−14	8	12

2	1	3	−12	−6	+20
		2	10	−4	−20
2	1	5	−2	−10	0
		2	14	24	
	1	7	12	12	

$f(2)=0$ so 2 is a solution and $f(x) = (x-2)(x^3+5x^2-2x-10)$

so 2 is only a solution of mult. 1

Now find solutions to $x^3+5x^2-2x-10$ by trial and error.

−2	1	5	−2	−10
		−2	−6	+16
	1	3	−8	6

+4	1	5	−2	−10
		4	36	136
	1	9	34	126

−4	1	5	−2	−10
		−4	−4	24
	1	1	−6	14

5	1	5	−2	−10
		5	50	240
	1	10	48	230

−5	1	5	−2	−10
		−5	0	+10
	1	0	−2	0

so −5 is a solution

$$f(x) = (x-2)(x-(-5))(x^2-2)$$
$$= (x-2)(x+5)(x^2-2)$$

to get last 2 solutions set $f(x)=0$

$$(x-2)(x+5)(x^2-2)=0$$

$x-2=0$	$x+5=0$	$x^2-2=0$
$+2 \quad +2$	$-5 \quad -5$	$+2 \quad +2$
$x=2$	$x=-5$	$\sqrt{x^2}=\sqrt{2}$
		$x=\pm\sqrt{2}$

$$\{2, -5, +\sqrt{2}, -\sqrt{2}\}$$

7. Use Theorem on Rational Zeros of a Polynomial to find all solutions of

$$f(x) = 2x^5 + 5x^4 - 9x^3 - 18x^2 = 0$$

$$f(x) = x^2 \underbrace{(2x^3 + 5x^2 - 9x - 18)}_{\text{Use theorem on}} = 0$$

$$2x^3 + 5x^2 - 9x - 18$$

possible values of c are factors of −18

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

possible values of d are factors of 2

$\pm 1, \pm 2$

possible $c/d = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \pm \frac{3}{2}, \cancel{\pm \frac{6}{2}}, \pm \frac{9}{2}, \cancel{\pm \frac{18}{2}}$

By trial & error

1	2	5	−9	−18
		2	7	−2
	2	7	−2	−20
−1	2	5	−9	−18
		−2	−3	+12
	2	3	−12	−6
2	2	5	−9	−18
		4	18	18
	2	9	9	0

$f(2)=0$ so 2 is a solution and

$$f(x) = x^2(x-2)\underbrace{(2x^2+9x+9)}_{\text{now we can factor this}}$$

$2 \cdot 9 = 18$

$6 \cdot 3$

$$2x^2+9x+9$$
$$= 2x^2+6x+3x+9$$
$$= 2x(x+3)+3(x+3)$$
$$= (2x+3)(x+3)$$

so $f(x) = x^2(x-2)(2x+3)(x+3) = 0$

$x = 0$ (mult. 2), $x-2=0$, $2x+3=0$, $x+3=0$

$x - 2 = 0$; $+2 \ \ +2$; $x = 2$

$2x + 3 = 0$; $-3 \ \ -3$; $\frac{2x}{2} = \frac{-3}{2}$

$x + 3 = 0$; $-3 \ \ -3$; $x = -3$

The Solutions to f(x) are $\{0 \text{ (mult. 2)}, 2, -\frac{3}{2}, -3\}$

8. Find the domain of $f(x) = \frac{x+2}{x^3-9x}$

$$x^3-9x \neq 0$$
$$x(x^2-9) \neq 0$$
$$x(x-3)(x+3) \neq 0$$

$x \neq 0$ $\quad$ $x-3 \neq 0$ $\quad$ $x+3 \neq 0$

$\quad\quad\quad\quad$ $+3 \;\; +3$ $\quad\quad$ $-3 \;\; -3$

$\quad\quad\quad\quad$ $x \neq 3$ $\quad\quad\quad$ $x \neq -3$

$$D = (-\infty,-3) \cup (-3,0) \cup (0,3) \cup (3,\infty)$$

9. Find the domain of $f(x) = \frac{\sqrt{3x+2}}{x^2-4}$

Cond. 1

$$3x+2 \geq 0$$
$$-2 \quad -2$$
$$\frac{3x}{3} \geq \frac{-2}{3}$$
$$x \geq -2/3$$

Cond 2

$$x^2-4 \neq 0$$
$$(x-2)(x+2) \neq 0$$

$x-2 \neq 0$ $\quad$ $x+2 \neq 0$

$+2 \;\; +2$ $\quad\quad$ $-2 \;\; -2$

$x \neq 2$ $\quad\quad$ $x \neq -2$

$$D = [-2/3, 2) \cup (2,\infty)$$

10. Find the domain of $f(x) = \frac{x-1}{\sqrt{x+3}}$.

$$x+3 > 0$$
$$-3 \quad -3$$
$$x > -3$$

(note that $x+3$ cannot equal zero since it is in the denominator)

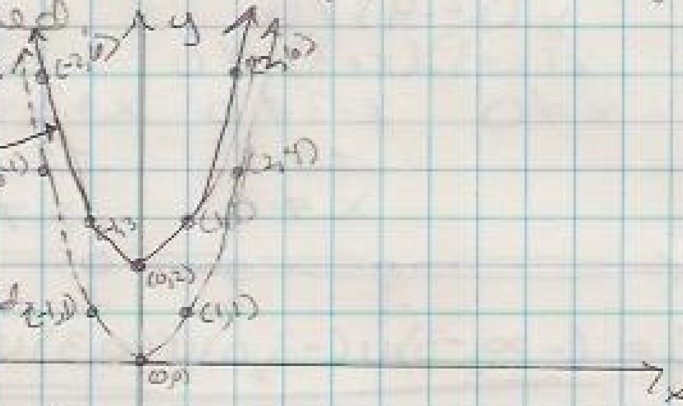
$$D = (-3,\infty)$$

11. Sketch the graph of $f(x) = x^2 + 2$ making use of symmetry, shifting, stretching, compressing, or reflecting as needed.

first graph $g(x) = x^2$

now graph $f(x) = g(x) + 2 = x^2 + 2$ by shifting $g(x)$ up 2 units.

Add 2 to each y-coord.

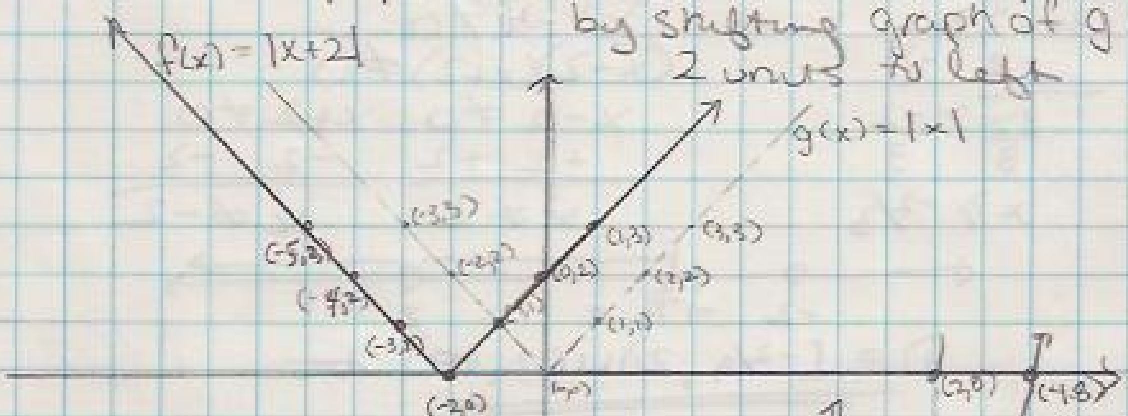


12. Sketch $f(x) = |x+2|$

graph $g(x) = |x|$

then graph $f(x) = g(x+2) = |x+2|$

by shifting graph of $g(x)$ 2 units to left



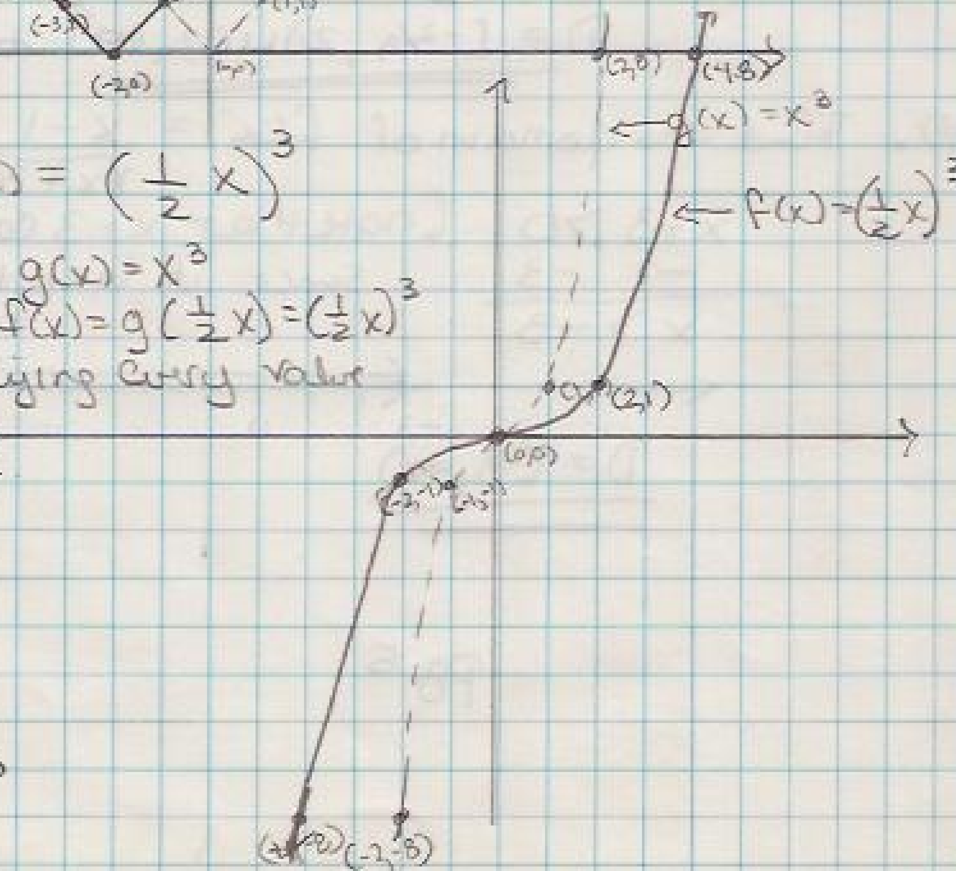
13. Sketch $f(x) = \left(\frac{1}{2}x\right)^3$

First sketch $g(x) = x^3$

Then graph $f(x) = g\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^3$

by multiplying every value of x by

$\frac{1}{\frac{1}{2}} = 2$



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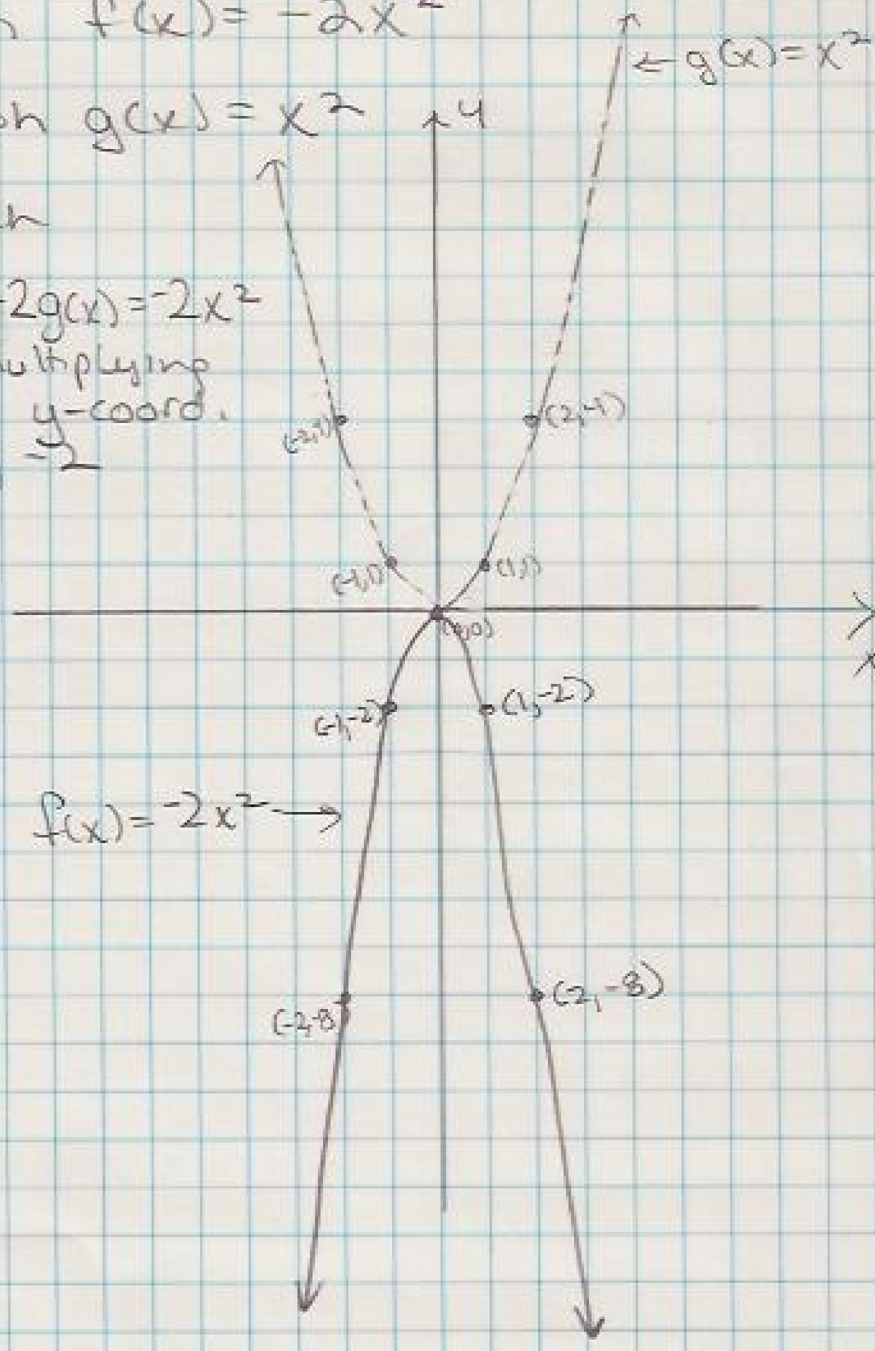
14. Sketch $f(x) = -2x^2$

First graph $g(x) = x^2$

Then graph

$f(x) = -2g(x) = -2x^2$

by multiplying each y-coord. by -2



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