MATH 110 (Gwinn-Edwards) Section #05
Fall 2008 Final
December 12, 2008

SOLUTIONS

Show all your work for full credit.

#1 Given the polynomial $f(x)=\underbrace{2x^4-3x^3}_{1}\underbrace{-12x^2+7x}_{2}+6$

(a) Use Descartes Rule of Signs to determine the number of possible positive real, negative real, and nonreal complex zeros of f(x). (The following table is provided for you to show your answer)

for negative real zeros
$f(-x)=2(-x)^4-3(-x)^3-12(-x)^2+7(-x)+6$
$= \underbrace{2x^4+3x^3-12x^2}_{1}\underbrace{-7x+6}_{2}$

# positive real zeros 2 or 0	2	2	0	0
# negative real zeros 2 or 0	2	0	2	0
# nonreal complex zeros	0	2	2	4
Total # of zeros	4	4	4	4

(b) Use the Theorem of Rational Zeros to find all the zeros of f(x). If you need extra room please work on the back of the previous page. (or this page, whatever ☺)

List all the possible zeros of f(x) $\pm 1/2, \pm 1, \pm 3/2, \pm 2, \pm 3, \pm 6$

possible values of c are factors of 6; $\pm 1, \pm 2, \pm 3, \pm 6$
" " " d " " " 2 $\pm 1, \pm 2$

$\frac{c}{d} = \frac{\pm 1}{1}, \frac{\pm 2}{1}, \frac{\pm 3}{1}, \frac{\pm 6}{1}, \frac{\pm 1}{2}, \cancel{\frac{\pm 2}{2}}, \frac{\pm 3}{2}, \cancel{\frac{\pm 6}{2}}$

1 | 2 -3 -12 7 6
2 -1 -13 -6
2 -1 -13 -6 | 0

so 1 is a zero of f

-1 | 2 -1 -13 -6
-2 3 +10
2 -3 -10 | 4

and $f(x) = (x-1)(x+2)(2x^2-5x-3)$

2 | 2 -1 -13 -6
4 6 -14
2 3 -7 | -20

factor this
$2(-3) = -6$
$-6+1$

-2 | 2 -1 -13 -6
-4 10 6
2 -5 -3 | 0

so -2 is a zero of f

List the actual zeros of f(x): {1, -2, -1/2, 3}

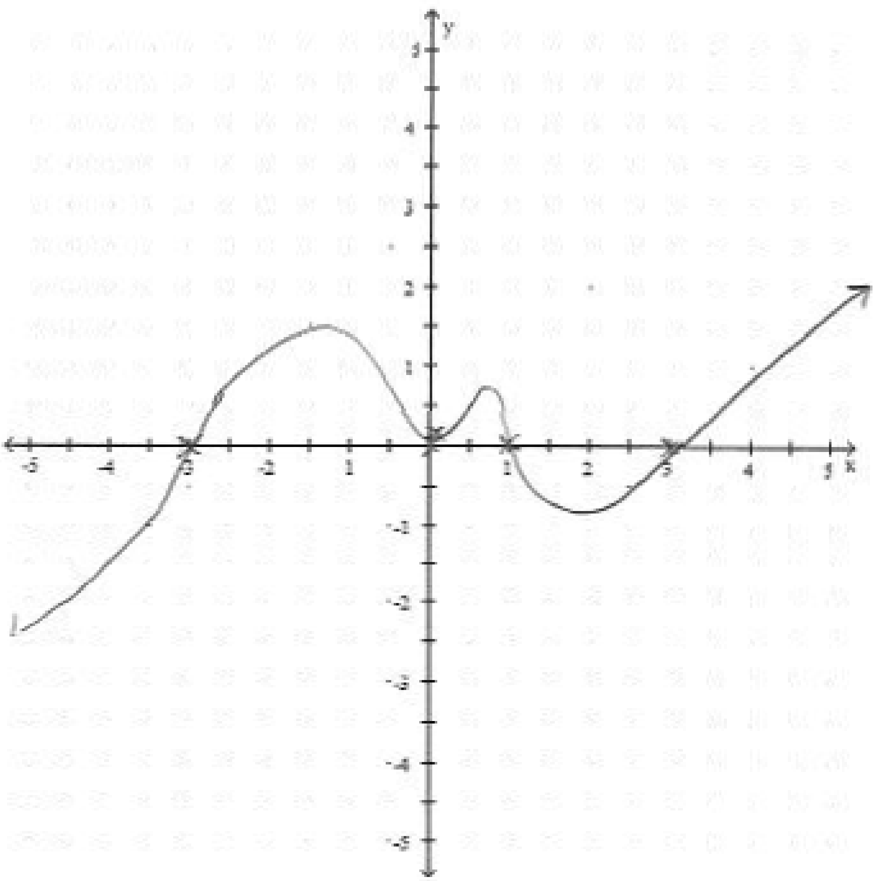
$2x^2-5x-3=0$
$2x^2-6x+x-3$
$2x(x-3)+1(x-3)=0$
$(2x+1)(x-3)=0$
$2x+1=0 \quad x-3=0$
$x=-1/2 \quad x=3$

#2 Graph the function $f(x)=x^2(x-3)(x-1)(x+3)$

Zeros are $\{0 \text{ (mult 2)}, 3, 1, -3\}$

The following table is provided for your convenience.

	-4	-1	$1/2$	2	4
Intervals	$(-\infty,-3)$	$(-3,0)$	$(0,1)$	$(1,3)$	$(3,\infty)$
x^2	+	+	+	+	+
$x-3$	−	−	−	−	+
$x-1$	−	−	−	+	+
$x+3$	−	+	+	+	+
f(x)	−	+	+	−	+



#3 Sketch the graph of $f(x)=\dfrac{x-2}{x^2-2x-3}$. $= \dfrac{x-2}{(x-3)(x+1)}$

a) Find the x-intercept.

$x-2=0$
$x=2$
$(2,0)$

The x-intercept is (2 ,0).

b) Find the vertical asymptotes.

$x^2-2x-3=0$
$(x-3)(x+1)=0$
$x=3 \quad x=-1$

The vertical asymptotes are x = 3 and x = −1

c) Find the y-intercept. Let $x=0$

$f(0) = \dfrac{0-2}{0^2-2(0)-3} = \dfrac{-2}{-3} = \dfrac{2}{3}$

The y-intercept is (0, 2/3).

d) Find the horizontal asymptote.

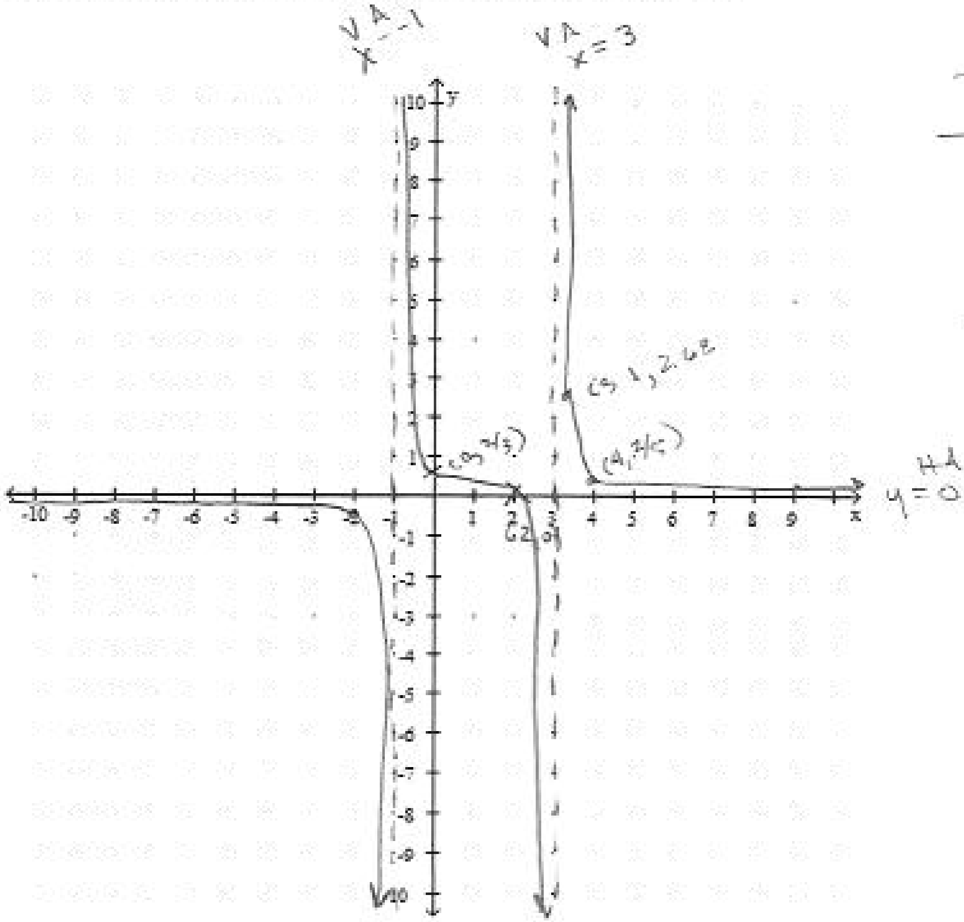
Since the degree of num < deg of den the HA is $y=0$

The horizontal asymptote is y = 0 .

e) Does the horizontal asymptote cross the graph of f ? If so, write the coordinates of this point.

Yes, at (2,0), we found this in step (a)

f) Sketch the function on the graph below Be sure to label your points on the graph, and find $f(x)$ for at least one value of x in each interval of the domain.



x	$\dfrac{x-2}{(x-3)(x+1)}$
4	$\dfrac{2}{(1)(5)} = 2/5$
9	$\dfrac{7}{6 \cdot 10} = 7/60$
3.1	$\dfrac{1.1}{(0.1)(4.1)} = \dfrac{1.1}{0.41} = 2.68$
−2	$\dfrac{-4}{-5(-1)} = -4/5$

#4 Sketch the graph of $f(x) = \dfrac{x^2 - 2x - 8}{x^2 + x - 6} = \dfrac{(x-4)(x+2)}{(x+3)(x-2)}$

a) Find the x-intercepts

$(x-4)(x+2) = 0$

$x = 4$ or $x = -2$

The x-intercept is (4,0) and (-2,0)

b) Find the vertical asymptotes.

$(x+3)(x-2) = 0$

$x = -3$ or $x = 2$

The vertical asymptotes are x = -3 and x = 2

c) Find the y-intercept. Let $x = 0$

$f(0) = \dfrac{0^2 - 2(0) - 8}{0^2 + 0 - 6} = \dfrac{-8}{-6} = \dfrac{4}{3}$

The y-intercept is $(0, \frac{4}{3})$.

d) Find the horizontal asymptote.

since deg num = deg den

then $y = \frac{1}{1} = 1$ is the HA

The horizontal asymptote is y = 1.

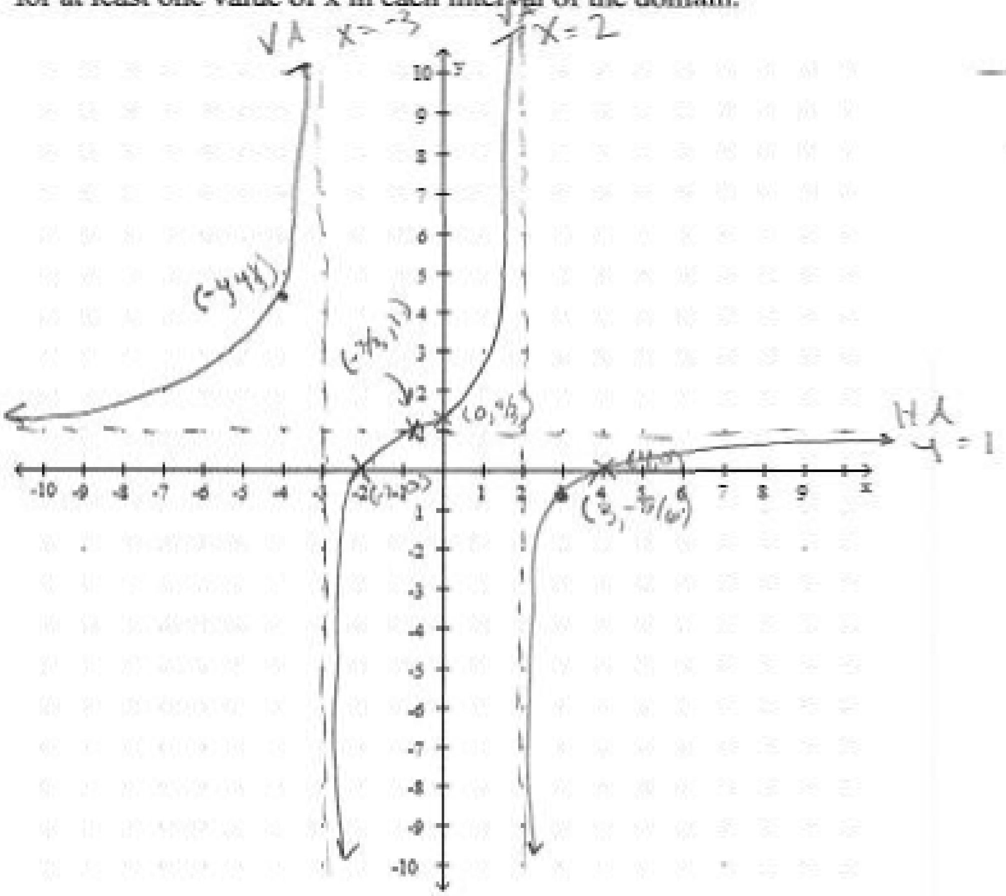
e) Does the horizontal asymptote cross the graph of f? If so, write the coordinates of this point.

$1 = \dfrac{x^2 - 2x - 8}{x^2 + x - 6} \Rightarrow x^2 + x - 6 = x^2 - 2x - 8$

$3x = -2 \rightarrow x = -2/3$

$(-2/3, 1)$

f) Sketch the function on the graph below Be sure to label your points on the graph, and find $f(x)$ for at least one value of x in each interval of the domain.



x	$\frac{(x-4)(x+2)}{(x+3)(x-2)}$
3	$\frac{(-1)(5)}{(6)(1)} = \frac{-5}{6}$
-4	$\frac{(-8)(-2)}{(-1)(-6)} = \frac{16}{6} = \frac{13}{3} = 4\frac{1}{3}$

#5 What is the domain, D, of $f(x)=\dfrac{\sqrt{2x+3}}{x^2+x-6}$? (be sure to write the domain in interval notation).

Condition 1

$2x+3 \geq 0$

$2x \geq -3$

$x \geq -3/2 = -1\frac{1}{2}$

Condition 2:

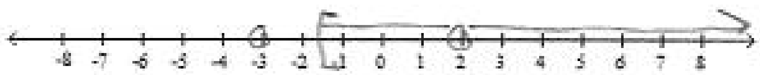
$x^2+x-6 \neq 0$

$(x+3)(x-2) \neq 0$

$x+3 \neq 0 \quad x-2 \neq 0$

$x \neq -3 \quad x \neq 2$

Graph the domain on the number line.



D = $[-3/2, 2) \cup (2, \infty)$

#6 Find the inverse function of $f(x)=\dfrac{2x-5}{3x+1}$ Assume f is one-to-one Show that $f(f^{-1}(x))=x$ and $f^{-1}(f(x))=x$

$y = \dfrac{2x-5}{3x+1}$

$y(3x+1) = 2x-5$

$3xy + y = 2x-5$

$\quad -y \quad -y$

$3xy = 2x - y - 5$

$\quad -2x \quad -2x$

$3xy - 2x = -y - 5$

$x(3y-2) = -y-5$

$x = \dfrac{-y-5}{3y-2}$

$$\boxed{f^{-1}(x) = y = \frac{-x-5}{3x-2} = \frac{x+5}{2-3x}}$$

$f(f^{-1}(x)) = f\left(\dfrac{x+5}{2-3x}\right) = \dfrac{2\left(\frac{x+5}{2-3x}\right)-5}{3\left(\frac{x+5}{2-3x}\right)+1} = \dfrac{\frac{2x+10-5(2-3x)}{2-3x}}{\frac{3x+15+1(2-3x)}{2-3x}}$

$= \dfrac{\frac{2x+10-10+15x}{2-3x}}{\frac{3x+15+2-3x}{2-3x}} = \dfrac{\frac{17x}{2-3x}}{\frac{17}{2-3x}}$

$= \dfrac{\cancel{17}x}{\cancel{2-3x}} \cdot \dfrac{\cancel{2-3x}}{\cancel{17}} = x$ ✓

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{2x-5}{3x+1}\right) = \dfrac{\frac{2x-5}{3x+1}+5}{2-3\left(\frac{2x-5}{3x+1}\right)}$

$\leftarrow f^{-1}(x) \quad = \dfrac{\frac{2x-5+5(3x+1)}{3x+1}}{\frac{2(3x+1)-6x+15}{3x+1}} = \dfrac{\frac{2x-5+15x+5}{3x+1}}{\frac{6x+2-6x+15}{3x+1}}$

$= \dfrac{\frac{17x}{3x+1}}{\frac{17}{3x+1}} = \dfrac{\cancel{17}x}{\cancel{3x+1}} \cdot \dfrac{\cancel{3x+1}}{\cancel{17}} = \underline{x}$ ✓

#7 Express $f(x)=3x^2-12x+17$ in the form $f(x)=a(x-h)^2+k$ using both the complete the square method and the theorem for locating the vertex of a parabola.

$$f(x) = 3x^2 - 12x + 17 - 17$$
$$-17$$

$$\frac{f(x)-17}{3} = \frac{3x^2-12x}{3}$$

$$? = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

$$\frac{f(x)-17}{3} + ? = x^2 - 4x + ?$$

$$\frac{f(x)-17}{3} + 4 = x^2 - 4x + 4$$
$$= (x-2)^2$$

$$-4 \qquad -4$$

$$3\left[\frac{f(x)-17}{3}\right] = 3\left[(x-2)^2 - 4\right]$$

$$f(x) - 17 = 3(x-2)^2 - 12$$
$$+17 \qquad +17$$

$$\underline{\underline{f(x) = 3(x-2)^2 + 5}}$$

$$f(x) = a(x-h)^2 + k, \quad a = 3, b = -12, c = 17$$

$$h = \frac{-b}{2a} = \frac{-(-12)}{2(3)} = \frac{12}{6} = 2$$

$$k = f(h) = f(2) = 3(2)^2 - 12(2) + 17$$
$$= 3(4) - 24 + 17$$
$$= 12 - 24 + 17$$
$$= -12 + 17$$
$$= 5$$

So $\underline{\underline{f(x) = 3(x-2)^2 + 5}}$ ✓

equal so we are confident we are correct.

#8 (a) Use the quadratic equation to find the zeros of f, (b) find the maximum or minimum value of f(x), and (c) sketch the graph of f (label your x-axis and y-axis intercepts and your vertex coordinates)

$f(x)=2x^2-8x+5$, $a=2,\ b=-8,\ c=5$

(a) Zeros of f:

Zeros of f are $\{3.225, 0.775\}$

$\dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-(-8) \pm \sqrt{(-8)^2-4(2)(5)}}{2(2)} = \dfrac{8 \pm \sqrt{64-40}}{4} = \dfrac{8 \pm \sqrt{24}}{4} = \dfrac{8}{4} \pm \dfrac{\sqrt{24}}{4} = 2 \pm \dfrac{\sqrt{4 \cdot 6}}{4} = 2 \pm \dfrac{2\sqrt{6}}{4} = 2 \pm \dfrac{\sqrt{6}}{2} = 2 \pm 1.225 = 3.225,\ 0.775$

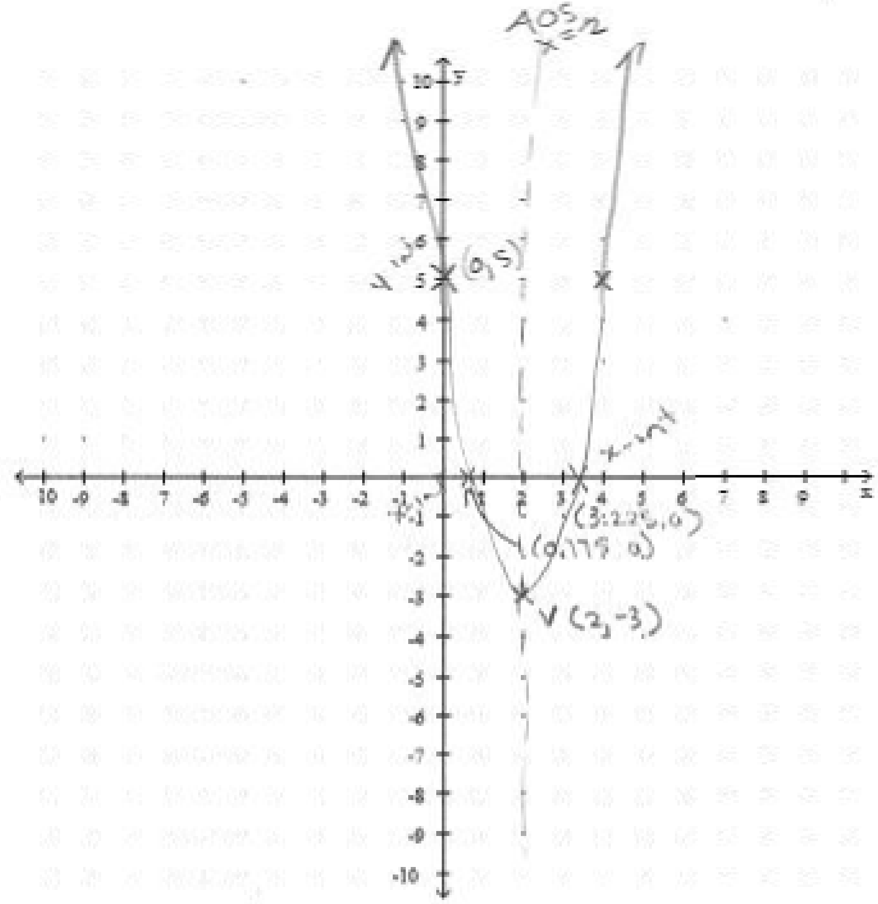
(b) Minimum or Maximum value of f:

$h = \dfrac{-b}{2a} = \dfrac{-(-8)}{2(2)} = \dfrac{8}{4} = 2$

$k = f(h) = f(2) = 2(2)^2 - 8(2) + 5 = 8 - 16 + 5 = -8 + 5 = -3$ ← min value of f since $a > 0$

(c) Sketch the parabola.

Write coordinates of x-intercepts (3.225,0), (0.775,0), y-intercepts (0,5), Vertex (2,-3)



y-int, let x = 0

$f(0) = 2(0)^2 - 8(0) + 5$

$= 5$

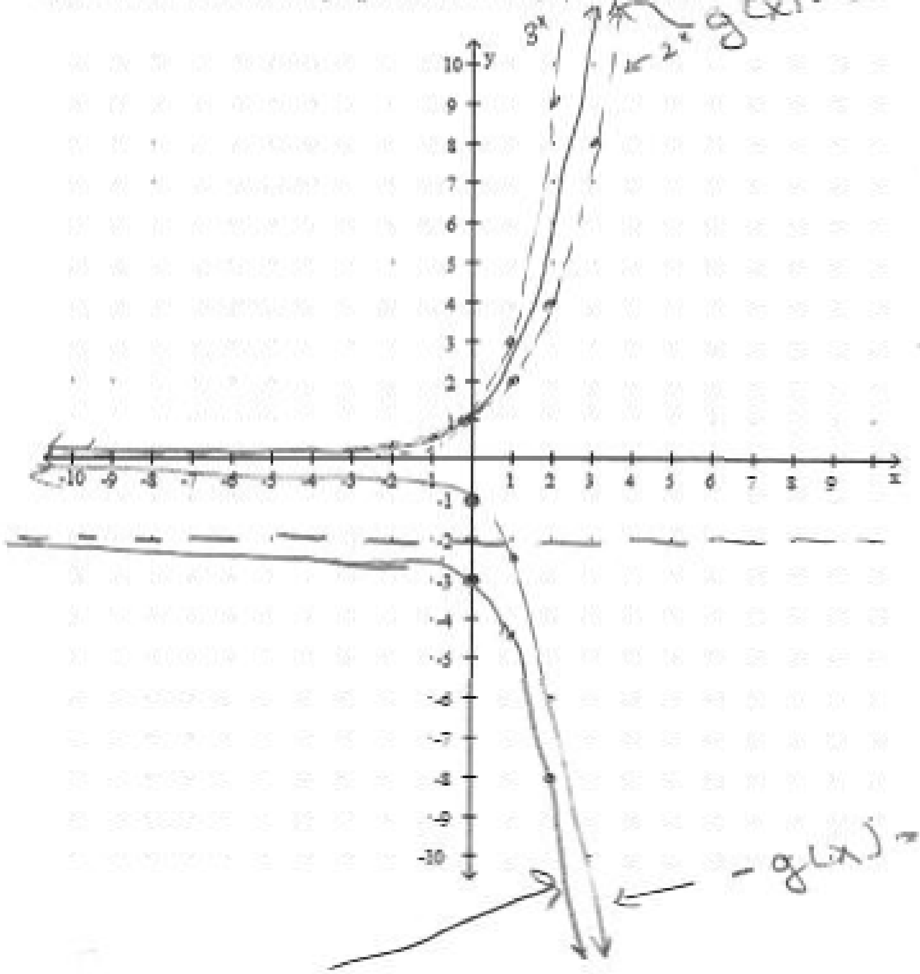
$(0,5)$

#9 (a) Sketch the approximate graph of $f(x)=e^x$ by first dashing in graphs of two other exponential functions that the final graph will fall between. (label the final graph clearly)

(b) On the same graph sketch $g(x)=e^{x-3}$ (label the horizontal asymptote).

(c) Also on the same graph sketch $h(x)=-e^x-2$

$g(x) = e^x$



x	2^x
0	1
1	2
2	4
3	8
-1	1/2
-2	1/4
-3	1/8

x	3^x
0	1
1	3
2	9
-1	1/3
-2	1/4

HA $y = -2$

$-g(x) = -e^x$

$-g(x) - 2 = -e^x - 2 = f(x)$