Solutions to MATH 110-17 Midterm 2 Review

2. Use I.V.T. to show that f has a zero between 2 & $\frac{5}{2}$.

$f(x) = 4x^4 - 9x^3 - 4x + 9$

$f(2) = 4(2)^4 - 9(2)^3 - 4(2) + 9 = 4 \cdot 16 - 9 \cdot 8 - 8 + 9$

$= 64 - 72 - 8 + 9 = -8 - 8 + 9 = -16 + 9 = -7 < 0$

$f(\frac{5}{2}) = 4(\frac{5}{2})^4 - 9(\frac{5}{2})^3 - 4(\frac{5}{2}) + 9 = 4\left(\frac{625}{16}\right) - 9\left(\frac{125}{8}\right) - \frac{20}{2} + 9$

$= \frac{625}{4} - \frac{1125}{8} - 10 + 9 = \frac{625 \cdot 2}{4 \cdot 2} - \frac{1125}{8} - \frac{1 \cdot 8}{1 \cdot 8} = \frac{1250 - 1125 - 8}{8}$

$= \frac{125 - 8}{8} = \frac{117}{8} > 0$

$f(2) = -7 < 0$ and $f(\frac{5}{2}) = \frac{117}{8} > 0$

Therefore by I.V.T. there exists a value of x, say c, with $2 < c < \frac{5}{2}$ such that $f(c) = 0 \Rightarrow c$ is a zero of f.

4. (a) $f(x) = \frac{5x+3}{3x-7}$

V.A. $3x - 7 = 0$

$3x = 7$

$x = 7/3 \leftarrow$ VA

H.A. deg num = deg den so $y = \frac{5}{3}$ is the H.A.

x-int $5x + 3 = 0$

$5x = -3$

$x = -3/5$

$(-3/5, 0) \leftarrow$ x-int

y-int $f(0) = \frac{5(0)+3}{3(0)-7}$

$= \frac{3}{-7} = -\frac{3}{7}$

$(0, -3/7)$ y-int

find where f crosses H.A. $\frac{5}{3} = \frac{5x+3}{3x-7}$

$5(3x-7) = (5x+3)3$

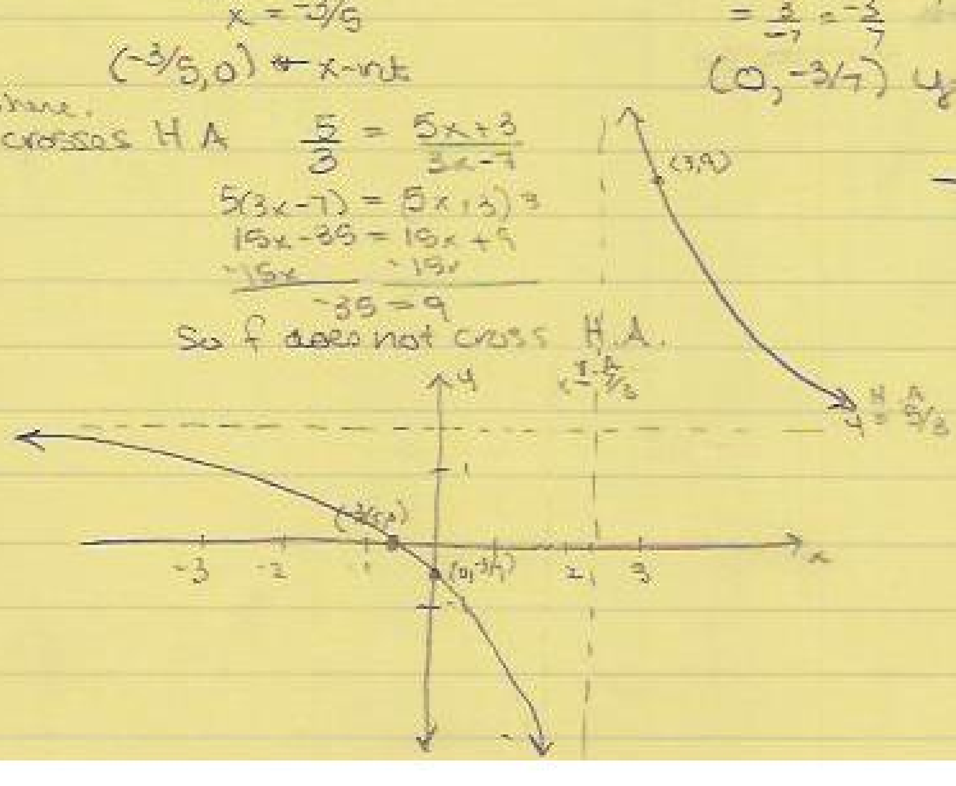
$15x - 35 = 15x + 9$

$-15x \quad -15x$

$-35 = 9$

So f does not cross H.A.

x	$\frac{5x+3}{3x-7}$
3	$\frac{15+3}{9-7} = \frac{18}{2} = 9$



4(b) $f(x) = \dfrac{x^2-3x-4}{x^2+x-6} = \dfrac{(x-4)(x+1)}{(x-2)(x+3)}$

V.A. $(x-2)(x+3)=0$

$x-2=0 \quad x+3=0$

$x=2,\ x=-3 \leftarrow$ V.A

H.A. deg num = deg den

so $y = \frac{1}{1} = 1$ is H.A.

x-int $(x-4)(x+1)=0$

$x-4=0 \quad x+1=0$

$x=4 \quad x=-1$

$(4,0)$ and $(-1,0)$ are x-intercepts

y-int

$f(0) = \dfrac{0^2-3(0)-4}{0^2+0-6} = \dfrac{-4}{-6} = \dfrac{4}{6} = \dfrac{2}{3}$

$(0, 2/3) \leftarrow$ y-int

point f crosses H.A.

$1 = \dfrac{x^2-3x-4}{x^2+x-6} \Rightarrow x^2+x-6 = x^2-3x-4$

$\underline{-x^2+3x \qquad -x^2+3x}$

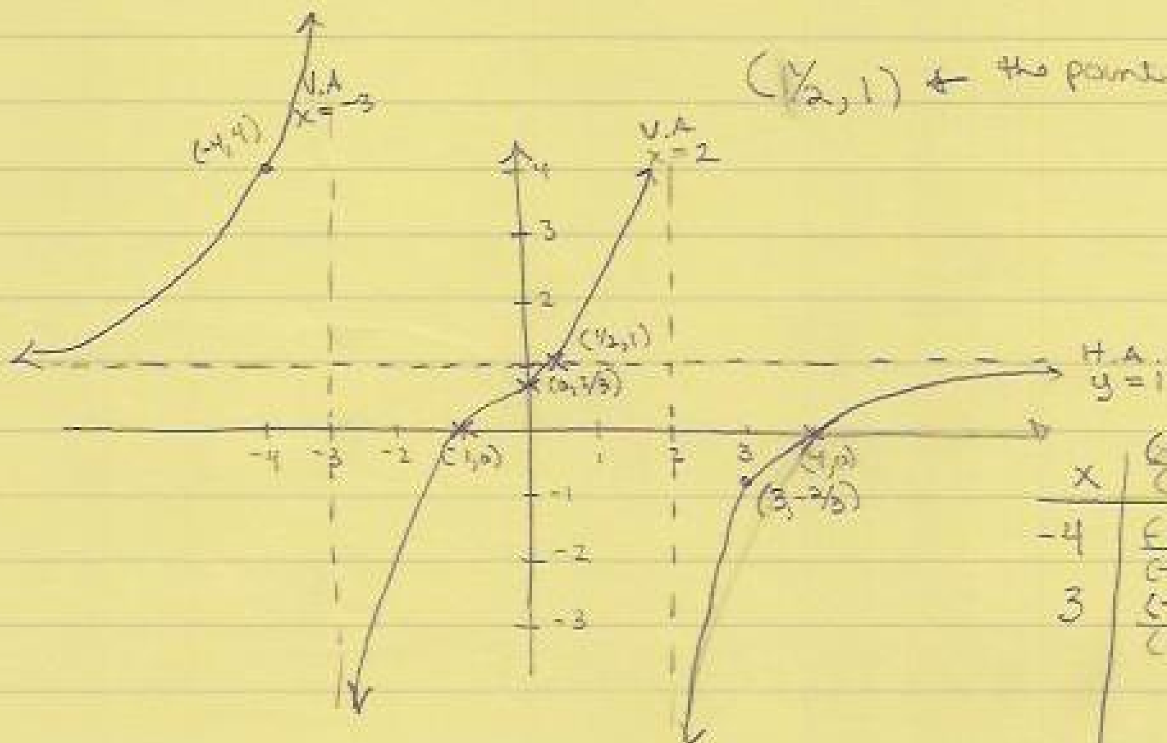
$4x-6 = -4$

$\underline{+6 \qquad +6}$

$4x = 2$

$x = 2/4 = 1/2$

$(1/2, 1) \leftarrow$ the point f crosses $y=1$.



x	$\dfrac{(x-4)(x+1)}{(x-2)(x+3)}$
-4	$\dfrac{(-8)(-3)}{(-6)(-1)} = \dfrac{24}{6} = 4$
3	$\dfrac{(-1)(4)}{(1)(6)} = \dfrac{-4}{6} = -\dfrac{2}{3}$

5. a) $f(x) = \frac{x}{x-2}$, $g(x) = \frac{3x}{x+4}$

i. $(f+g)(x) = f(x) + g(x) = \frac{x}{x-2} + \frac{3x}{x+4} = \frac{x(x+4)}{(x-2)(x+4)} + \frac{3x(x-2)}{(x+4)(x-2)}$

$= \frac{x^2+4x+3x^2-6x}{(x-2)(x+4)} = \frac{4x^2-2x}{(x-2)(x+4)} = \underline{\frac{2x(2x-1)}{(x-2)(x+4)}}$

$(f-g)(x) = f(x) - g(x) = \frac{x}{x-2} - \frac{3x}{x+4} = \frac{x(x+4)-3x(x-2)}{(x-2)(x+4)}$

$= \frac{x^2+4x-3x^2+6x}{(x-2)(x+4)} = \frac{-2x^2+10x}{(x-2)(x+4)} = \underline{\frac{-2x(x-5)}{(x-2)(x+4)}}$

$(fg)(x) = f(x)g(x) = \frac{x}{(x-2)} \cdot \frac{3x}{(x+4)} = \underline{\frac{3x^2}{(x-2)(x+4)}}$

$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x}{x-2}}{\frac{3x}{x+4}} = \frac{x}{x-2} \cdot \frac{x+4}{3x} = \underline{\frac{x+4}{3(x-2)}}$

ii. $D_{f+g, f-g, fg} = D_f \cap D_g = [(-\infty, 2) \cup (2, \infty)] \cap [(-\infty, -4) \cup (-4, \infty)]$

$\underline{D_f \cap D_g = (-\infty, -4) \cup (-4, 2) \cup (2, \infty)}$

iii. $D_{f/g} = D_f \cap D_g$ minus zeros of g

$= (-\infty, -4) \cup (-4, 2) \cup (2, \infty)$ minus $\{0\}$

$= \underline{(-\infty, -4) \cup (-4, 0) \cup (0, 2) \cup (2, \infty)}$

$g(x) = \frac{3x}{x+4} = 0$

$3x = 0$

$x = 0 \leftarrow$ zero of g

5(b) $f(x) = \frac{x+2}{x-1}$, $g(x) = \frac{x-5}{x+4}$

i. $(f\circ g)(x) = f(g(x)) = f\left(\frac{x-5}{x+4}\right) = \dfrac{\frac{x-5}{x+4}+2}{\frac{x-5}{x+4}-1}$

$$= \dfrac{\frac{x-5}{x+4} + \frac{2(x+4)}{1(x+4)}}{\frac{x-5}{x+4} - 1\frac{(x+4)}{(x+4)}} = \dfrac{\frac{x-5+2(x+4)}{x+4}}{\frac{x-5-(x+4)}{x+4}}$$

$$= \dfrac{\frac{x-5+2x+8}{x+4}}{\frac{x-5-x-4}{x+4}} = \dfrac{\frac{3x+3}{x+4}}{\frac{-9}{x+4}}$$

$$= \frac{3x+3}{\cancel{x+4}} \cdot \frac{\cancel{x+4}}{-9} = \frac{{}^1\cancel{3}(x+1)}{-\cancel{9}3} = \frac{(x+1)}{-3}$$

$(f\circ g)(x) = -\dfrac{x+1}{3}$ ←

$D_{f\circ g} = D_g$ minus values of x s.t. g(x) is not in D_f.

$D_g = (-\infty,-4)\cup(-4,\infty)$

D_f does not include 1 so

$g(x) \neq 1$

$\frac{x-5}{x+4} \neq 1$

$\Rightarrow x-5 \neq x+4$

$\quad -x \qquad -x$

$-5 \neq 4$

no values of x will be excluded by this.

so $\underline{\underline{D_{f\circ g} = (-\infty,-4)\cup(-4,\infty)}}$

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5(b) ii $(g\circ f)(x) = g(f(x)) = g\left(\frac{x+2}{x-1}\right)$

$$= \frac{\frac{x+2}{x-1} - 5}{\frac{x+2}{x-1} + 4} = \frac{\frac{x+2-5(x-1)}{x-1}}{\frac{x+2+4(x-1)}{x-1}} = \frac{\frac{x+2-5x+5}{x-1}}{\frac{x+2+4x-4}{x-1}}$$

$$= \frac{\frac{-4x+7}{x-1}}{\frac{5x-2}{x-1}} = \frac{-4x+7}{\cancel{x-1}} \cdot \frac{\cancel{x-1}}{5x-2} = \underline{\underline{\frac{-4x+7}{5x-2}}} \leftarrow (g\circ f)(x)$$

$D_{g\circ f} = D_f$ minus values of x s.t. $f(x)$ is not in D_g

$D_f = (-\infty, 1) \cup (1, \infty)$ $\quad$ D_g excludes -4

$$\frac{x+2}{x-1} \neq -4$$
$$x+2 \neq -4(x-1)$$
$$x+2 \neq -4x+4$$
$$\underline{+4x \qquad +4x}$$
$$5x+2 = 4$$
$$\underline{-2 \qquad -2}$$
$$5x = 2$$
$$x = 2/5$$

so we must exclude 2/5 from the domain of $D_{g\circ f}$

$D_{g\circ f} = (-\infty, 2/5) \cup (2/5, 1) \cup (1, \infty)$

7.(a). $f(x) = \frac{1}{x+3}$ $\quad$ (1) f is 1-1

(2) $y = \frac{1}{x+3}$

$$y(x+3) = 1$$
$$xy + 3y = 1$$
$$xy = 1-3y$$
$$x = \frac{1-3y}{y}$$
$$f^{-1}(x) = y = \underline{\underline{\frac{1-3x}{x}}} \leftarrow f^{-1}(x)$$

(3) $f(f^{-1}(x)) = f\left(\frac{1-3x}{x}\right) = \frac{1}{\frac{1-3x}{x} + 3}$

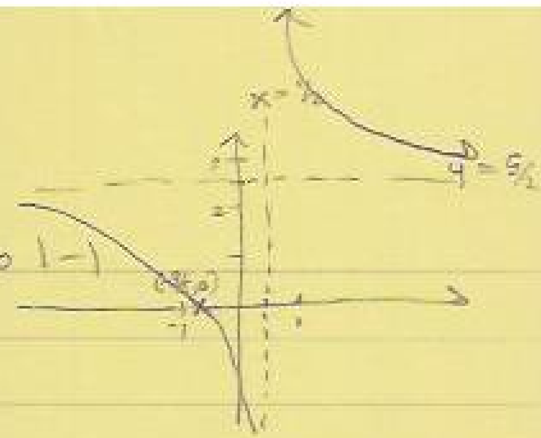
$$= \frac{1}{\frac{1-3x+3x}{x}} = \frac{1}{\frac{1}{x}} = \frac{1}{1} \cdot \frac{x}{1} = \underline{\underline{x}} \checkmark$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x+3}\right) = \frac{1-3\left(\frac{1}{x+3}\right)}{\frac{1}{x+3}}$$

$$= \frac{\frac{x+3}{x+3} - \frac{3}{x+3}}{\frac{1}{x+3}} = \frac{\frac{x+3-3}{x+3}}{\frac{1}{x+3}}$$

$$= \frac{x}{\cancel{x+3}} \cdot \frac{\cancel{x+3}}{1} = \underline{\underline{x}} \checkmark$$

7(b) $f(x) = \dfrac{5x+3}{2x-1}$ (1) f is 1-1



(2) $y = \dfrac{5x+3}{2x-1}$

$(2x-1)y = 5x+3$

$2xy - y = 5x+3$

$-5x+y \quad -5x+y$

$2xy - 5x = y+3$

$\dfrac{x(2y-5)}{2y-5} = \dfrac{y+3}{2y-5}$

$x = \dfrac{y+3}{2y-5}$

$f^{-1}(x) = y = \dfrac{x+3}{2x-5} \leftarrow f^{-1}(x)$

(3)

$f(f^{-1}(x)) = f\left(\dfrac{x+3}{2x-5}\right)$

$= \dfrac{5\left(\frac{x+3}{2x-5}\right)+3}{2\left(\frac{x+3}{2x-5}\right)-1}$

$= \dfrac{\frac{5x+15+3(2x-5)}{2x-5}}{\frac{2x+6-(2x-5)}{2x-5}}$

$= \dfrac{\frac{5x+15+6x-15}{2x-5}}{\frac{2x+6-2x+5}{2x-5}}$

$= \dfrac{\frac{11x}{2x-5}}{\frac{11}{2x-5}} = \dfrac{11x}{\cancel{2x-5}} \cdot \dfrac{\cancel{2x-5}}{\cancel{11}}$

$= x$ ✓

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{5x+3}{2x-1}\right) = \dfrac{\frac{5x+3}{2x-1}+3}{2\left(\frac{5x+3}{2x-1}\right)-5}$

$= \dfrac{\frac{5x+3+3(2x-1)}{2x-1}}{\frac{10x+6-5(2x-1)}{2x-1}} = \dfrac{\frac{5x+3+6x-3}{2x-1}}{\frac{10x+6-10x+5}{2x-1}}$

$= \dfrac{\frac{11x}{2x-1}}{\frac{11}{2x-1}} = \dfrac{\cancel{11}x}{\cancel{2x-1}} \cdot \dfrac{\cancel{2x-1}}{\cancel{11}} = x$ ✓