MATH 110-17 (Gwinn-Edwards)
Fall 2009
Midterm 1 Review

Understand the following theorems and how to apply them:

Remainder Theorem: If f(x) is divided by x-c, then the remainder will be f(c).

Factor Theorem: A polynomial has a factor x-c if and only if f(c) = 0.

Definition: **Zeros of a polynomial,** f(x), are solutions to f(x) = 0.

Complete Factorization Theorem for Polynomials. If f(x) has degree n>0, then there exist n numbers c_1, c_2, ... c_n, such that f(x) = a(x – c_1)(x-c_2)....(x-c_n), where a is the leading coefficient of f(x).

Descartes' Rule of Signs: If f(x) is a polynomial with real coefficients and a nonzero constant term.

(1) The number of positive real zeros of f(x) is either equal to the number of variations in sign of f(x) or less than that number by an even integer.
(2) The number of negative real zeros of f(+x) is either equal to the number of variations in sign of f(-x) or less than that number by an even integer.

Theorem on Conjugate Pair Zeros of a Polynomial: If f(x) of degree > 1 has real coefficients and $z=a+bi$ with $b\neq0$ is a complex zero of f(x), then the conjugate $\bar{z}=a-bi$ is also a zero of f(x).

Theorem on Rational Zeros of a Polynomial: If a polynomial has integer coefficients and if c/d is a rational zero of f(x) such that c and d have no common factors, then

(1) The numerator c of the zero is a factor of the constant term of f(x).
(2) The denominator d of the zero is a factor of the leading coefficient of f(x).

Standard Equation of a Parabola with Vertical Axis: The graph of $y=a(x-h)^2+k$ for $a\neq0$ is a parabola with vertex at (h,k) and vertical axis. The parabola opens up if a>0, and opens down if a<0.

Theorem of locating the vertex of a parabola: The vertex of a parabola $y=ax^2+bx+c$ has x coordinate $h=-\frac{b}{2a}$ and y-coordinate $k=f(-\frac{b}{2a})=f(h)$

Theorem on Maximum or Minimum Value of a Quadratic Function: For $f(x)=ax^2+bx+c\,,a\neq0$

$f(-\frac{b}{2a})$ is

(a) the maximum value of f if a<0,
(b) the minimum value of f if a>0.

Practice Problems:

1. Use remainder theorem to find $f(-3)$ if $f(x)=x^3+2x^2+3x+4$. Check your answer.

2. Find a polynomial f(x) with zeros at 0, -2, and 3, which satisfies the given condition f(1) = 30.

3. Show that 2 is a zero of f(x) of multiplicity 2 and express f(x) as a product of linear factors.
   $f(x)=3x^4-5x^3-12x^2+12x+16$

4. Use Descartes' Rule of Signs to determine the number of possible positive, negative, and nonreal complex solutions to the equation $f(x)=x^5-3x^4+2x^3-6x+3$

5. Find a polynomial with real coefficients and leading coefficient 1 having the given zeros and leading coefficient 1. Express f(x) as a product of linear and quadratic polynomials with real coefficients that are irreducible over the real numbers. Zeros are at: 3, 0, 2+i; degree 4.

6. Use Theorem on Rational Zeros of a Polynomial to find all the solutions of f(x).
   $f(x)=x^4+3x^3-12x^2-6x+20$

7. Use Theorem on Rational Zeros of a Polynomial to find all the solutions of f(x).
   $f(x)=2x^5+5x^4-9x^3-18x^2$

8. Find the domain of $f(x)=\frac{x+2}{x^3-9x}$

9. Find the domain of $f(x)=\frac{\sqrt{3x+2}}{x^2-4}$

10. Find the domain of $f(x)=\frac{x-1}{\sqrt{x+3}}$

11. Sketch the graph of $f(x)=x^2+2$ making use of symmetry, shifting, stretching, compressing, or reflecting as needed.

12. Sketch the graph of $f(x)=|x+2|$ (same instructions as 11)

13. Sketch the graph of $f(x)=(\frac{1}{2}x)^3$ (same instructions as 11)

14. Sketch the graph of $f(x)=-2x^2$ (same instructions as 11)

15. Sketch the piece-wise function defined as follows:

$$f(x) = \begin{cases} 2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ -x+1 & \text{if } x \ge 1 \end{cases}$$

16. Express f(x) in the form $f(x) = a(x-h)^2 + k$ use both the complete the square method and theorem for locating the vertex of a parabola.

$$f(x) = -x^2 - 6x - 5$$

17. Same instructions as 16.

$$f(x) = 2x^2 - 12x + 19$$

18. (a) Use the quadratic formula (or factor if possible) to find the zeros of f, (b) Find the maximum or minimum value of f(x), and (c) Sketch the graph of f.

$$f(x) = 6x^2 + 7x - 24$$

19. Same instructions as 18.

$$f(x) = -4x^2 + 4x - 1$$

20. Same instructions as 18.

$$f(x) = 2x^2 - 4x - 11$$